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DIFFERENCES IN THE ASSESSMENT OF PLASTIC COLLAPSE IN BS 7910:2005 AND R6/FITNET FFS PROCEDURES

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ABSTRACT

At present within the fracture assessment routes of different codes and standards, two different options for the assessment of plastic collapse, L_r, are available, namely reference stress and limit load approaches.

Recent comparative studies have shown significant differences in the assessment of plastic collapse depending on whether the reference stress solutions in BS 7910:2005 or the limit load solutions in R6/FITNET are used for the calculation of L_r.

In this paper, differences with respect to the choice of solutions and boundary conditions will be illustrated and observations regarding the route that the Codes should take with respect to a unified assessment will be discussed.

INTRODUCTION

BS 7910, the UK procedure for the assessment of flaws in metallic structures, which was first published (as PD6493) nearly 30 years ago, is currently being revised to prepare a new version intended to be released in 2012. The most far-reaching changes are being undertaken in the fracture assessment of BS 7910 in order to reflect advances made in recent years.

Assessment of plastic collapse, via the parameter L_r, is one of the core elements of a fracture assessment within the context of the Failure Assessment Diagram (FAD) approach, along with assessment of fracture via the parameter, K_r (the ratio of the applied linear elastic stress intensity factor to the material's fracture toughness). Hence, it deserves considerable attention and in this respect, an extensive literature survey was conducted to make recommendations for the new BS 7910.

Within the fracture assessment routes of different codes and standards, two different options for the assessment of L_r are available. While the current edition of BS 7910 adopts the reference stress approach [1], two other major procedures, the UK power industry's 'R6' procedure and the European fitness-for-service procedure FITNET, advise the limit load approach for the calculation of L_r [2]-[4].

However, recent comparative studies have shown significant differences in the assessment of plastic collapse depending on whether the reference stress solutions in BS 7910:2005 or the limit load solutions in R6/FITNET are used for the calculation of L_r.

Although ideally an assessment made according to either approach should arrive at similar results, it was found in the course of development of the new BS7910 procedure that BS 7910:2005 and R6/FITNET may deliver different results due to the different solutions chosen (from the many available in the literature).

In this paper, differences with respect to the choice of solutions and boundary conditions will be illustrated for two types of structural components, namely flat plates and pipes/cylinders, containing three different defect types; through-thickness, surface and embedded flaws. For pipes/cylinders, axially and circumferentially oriented flaws were analysed. Pure tension, pure bending and combined tension and bending loading conditions were considered for each case analysed.

Consequently, it will be shown that a simple analytical relationship between the current BS7910:2005 reference stress solutions and R6/FITNET limit load solutions is not straightforward and finally this will be followed with recommendations regarding plastic collapse assessment strategy of the new BS7910. Furthermore, a general survey summarizing features of the new BS 7910 and explaining adoption of new advanced fracture assessment procedures can also be found in [5], and this paper should be read alongside [5] and [26].

NOMENCLATURE

CTOD	=	crack tip opening displacement
FAD	=	Failure Assessment Diagram

FFS	=	Fitness-for-service
a	=	half flaw length for through-thickness flaw, flaw height for surface flaw or half height for embedded flaw
B	=	section thickness in plane of flaw
c	=	half flaw length for surface or embedded flaws
F	=	axial tensile load
F_e	=	generic term for yield limit load
k	=	normalised crack off-set
L_r	=	ratio of applied load to yield or proof load
L_r	=	measure of proximity to plastic collapse
L_r^b	=	normalised limit bending moment
L_r^N	=	normalized total axial limit load
M^b	=	applied bending moment
M_e^b	=	limit bending moment
P_b	=	primary bending stress
$P_{b,l}$	=	primary bending stress due to locally applied bending loads
P_m	=	primary membrane stress
$P_{m,a}$	=	primary membrane stress due to global axial loads
$P_{m,b}$	=	primary membrane stress due to global bending moments
$P_{m,p}$	=	primary membrane stress due to pressure loading
p	=	shortest distance from material surface to embedded flaw
p'	=	internal pressure
p_L	=	limit pressure for pipes/cylinders
R_e	=	yield strength, R_{eL} or $0.95R_{eH}$ MPa, for discontinuously yielding materials
r_i	=	inner radius of pipe/cylinder
r_m	=	mean radius of a pipe/cylinder, $r_m=(r_o+r_i)/2$
r_o	=	outer radius of pipe/cylinder
S_{a1}, S_{a2}	=	normalised maximum and minimum axial stresses respectively
t	=	wall thickness of pipe/cylinder
W	=	width of plate
α	=	a/t
α''	=	function of a, c, B and W used in calculation of collapse stresses
β	=	angle defining the neutral axis position
λ	=	load ratio for combined tension and bending
$\sigma_{n,b}$	=	bending component of collapse stress
$\sigma_{n,m}$	=	membrane component of collapse stress
σ_{ref}	=	reference stress
θ	=	parametric angle to identify position along an elliptic flaw front
χ	=	ratio between the axial load and pressure induced axial load

FRACTURE ASSESSMENT IN BS 7910, R6 AND FITNET

Among numerous published FFS procedures, for the assessment of fracture integrity, BS 7910, R6 and FITNET can be regarded as the most-commonly used European procedures and all three procedures offer generic rules/routes to cover a wide range of components and structures made of metallic materials. This generic approach is their main difference from other industry specific procedures.

Common features of fracture assessment to BS 7910, R6 and FITNET include assessment of plastic collapse L_r, material's resistance to fracture K_r and representation of those results on an FAD.

Whilst calculation of K_r is quite similar in all three procedures, the construction of the failure assessment curve in R6 and FITNET differs somewhat from that of current BS 7910 FAD. R6 and FITNET provide more detailed and explicit guidance on treatment of constraint and strength mismatch in fracture assessments. Both R6 and FITNET benefit from the structure of limit load solutions allowing incorporation of weld zone's material properties into construction of the failure assessment curve. Furthermore, in R6 and FITNET the same limit load solutions employed in the construction of failure assessment curve can be further used for the determination of L_r, whereas BS7910:2005 uses the reference stress solutions for the determination of L_r.

Although in general terms the differences in failure assessment curves of the three procedures do not lead to inconsistencies in the course of assessment, and ideally adoption of reference stress or limit load approaches for the assessment of L_r should not change the results, it was found that BS 7910:2005 and R6/FITNET may deliver different L_r values due to the different solutions chosen from the many available in the literature.

To this end, after the introduction of general assessment strategy of each procedure, the equations used to determine L_r in the three documents and their validity ranges will be presented and compared in detail.

The current edition of the BS 7910 offers three analysis routes designated as "Levels" depending on available material properties and failure mode. Level 1 is the quick screening method for the assessment, derived based on the CTOD design curve and later adapted to be used in conjunction with FAD, whereas Level 2 is a more sophisticated analysis using either generic or material specific (if stress-strain curve is available) FADs for initiation assessment; this requires a single characteristic fracture toughness value of CTOD, J or K. Further to that, a Level 2 analysis requires stresses to be expressed in terms of membrane and bending components. The aim of Level 3, which is quite similar to Level 2 but potentially more accurate, is assessment of tearing; it requires fracture toughness in terms of tearing resistance curves.

R6 contains also three levels of assessment, designated as Option 1, 2 and 3. However, the hierarchy of Options is differently designed. Although results of an assessment in all the three Options can be illustrated on FADs and Option 1 is similar to BS 7910's Level 2, Option 2 requires material specific tensile properties for the construction of material specific FADs. Option 3 delivers the most accurate results with the help of numerical analysis of crack driving force. Depending on the available fracture toughness data, both initiation and tearing failures can be assessed using any of the Options.

In FITNET, there is also a hierarchy of fracture assessment routes, designated as "Options" and varying from Option 0 to Option 5. The choice between different Options depends on the quality of input data as in the former procedures introduced above. FITNET's assessment philosophy and equations are very similar to R6's but it is structured differently.

Turning back to revision of BS7910, the respective British Standard (BS) committee decided that the new edition should harmonise with R6 and FITNET where possible and support use of more advanced analysis techniques, whilst preserving compatibility with previous editions of BS 7910 (unless the methods are obsolete or there is evidence that they are unsafe); this avoids the need to re-visit analyses carried out with an earlier edition of the procedure, and makes the document more amenable to returning and occasional users. Hence, for a more accurate analysis, the R6/FITNET FADs were adopted and a hybrid approach allowing usage of both reference stress and limit load solutions for the assessment of L_r was devised.

ASSESSMENT OF PLASTIC COLLAPSE, L_r, IN BS 7910, R6 AND FITNET

For the examination of elastic-plastic deformation and fracture behaviour of engineering structures, reference stress is a very powerful tool allowing the user to make predictions on the proximity of plastic collapse for a given geometry under given loading conditions. Several analytical methods like J-estimation approach and various numerical methods are available for the estimation of reference stresses [6].

However, limit load estimation has also emerged as an equally powerful tool for the assessment of proximity to plastic collapse. The main idea behind limit load solutions is that unrestricted plastic flow occurs when stresses in some section of the body are in general yield, thus providing a collapse mechanism and a limit to the load value. For the illustration of this mechanism, materials are assumed to be elastic-perfectly plastic during the development of limit load solutions.

Since in reality, structural response is neither purely brittle nor purely ductile but exhibits aspects of both, and metallic materials are strain hardenable, these properties are also considered with the usage of a failure assessment curve on a FAD and the calculated measure of proximity to plastic collapse is further evaluated together with other material properties in the context of a fracture assessment.

Turning back to the reference stress and limit load solutions, the principles for the derivation of reference stress and limit load solutions can be found in various compendia together with explicit solutions for different flaw types in many simple and complex geometries available before 1998 [7]-[9] covering major works like [10]-[13]. In recent years, more limit load solutions for plates, and for thin-walled and thick-walled pipes/cylinders have been published as a result of extensive research in the field [14]-[25].

In the context of a structural integrity analysis aiming to assess fracture resistance of a flawed structure, plastic collapse L_r can be assessed using reference stresses, σ_{ref}, as indicated in equation (1), by dividing it by the yield strength, R_e, of the material. Alternatively, proximity to plastic collapse can also be assessed using plastic limit loads, which are derived based on the reference stress J-estimation approach. If limit load solutions are to be used, L_r is the ratio of the total applied load giving rise to the primary stresses to the plastic limit load of the flawed structure, see also equation below.

$$L_r = \frac{\sigma_{ref}}{R_e} = \frac{\text{applied load}}{\text{limit load}} \tag{1}$$

where applied load can be tension, bending or pressure. For these loading cases L_r is calculated using the equations (1a)-(1c) respectively

$$L_r = \frac{F}{F_e^N} \tag{1a}$$

$$L_r = \frac{M^b}{M_e^b} \tag{1b}$$

$$L_r = \frac{p'}{p_L} \tag{1c}$$

However, when welded structures having dissimilar material properties are to be analysed using the reference stress approach in BS7910:2005, the only modification which can be made in equation (1) is to replace the yield strength of the base material with that of the weld material's. In order to increase the accuracy of such an analysis, the material properties of the welds should be incorporated into the formulations in a more structured manner. To this end, the main advantage of limit load solutions is their structure, allowing incorporation of material strength mismatch and the relative location of the crack in the mismatched region, which makes the concept of limit load easier to understand and use. Hence, the vast majority of new solutions, including those dealing with mismatch, are expressed as limit loads rather than reference stresses.

Before discussing the optimum assessment strategy for weldments, turning back to the assessment of base materials, it was found that the L_r values calculated in accordance with BS 7910:2005 using reference stresses and those derived from R6/FITNET using limit loads were often significantly different,

$$L_r^{BS7910} = \frac{\sigma_{ref}}{R_e} \neq \frac{\text{applied load}}{\text{limit load}} = L_r^{R6\ /\ FITNET} \tag{2}$$

Prior to handling more complex structures, priority should be given to the identification of the extent of this difference. Hence, two most common geometries, flat plates and pipes/cylinders containing through-thickness, surface and embedded flaws were assessed, see also Table 1 for an overview of the analysis matrix. For pipes/cylinders, axially and circumferentially oriented flaws were considered separately.

The analytical procedure of calculating L_r in accordance with BS7910:2005 and R6/FITNET for a given flaw type was performed using the commercial softwares MatLab and MathCad. Details of each analysis can be found in the following sections representing results.

ASSESSMENT OF FLAT PLATES CONTAINING FLAWS

Flat plates constitute the first group of assessments. For the examination of flat plates, three different flaw types were considered. The geometrical definition of through-thickness, surface and embedded flaws can be seen in Figures 1-3 respectively.

Since the structure of the reference stress solutions expressing applied loads as membrane stress P_m and bending stress P_b (see Annex A), and the limit load solutions expressing the applied loads as normal forces F and bending moments M^b (see Annex B), are different, the following conversion formulae for flat plates were used in the calculations:

$$F = P_m WB \tag{3}$$

$$P_b = \left(\frac{6}{WB^2}\right) M^b \tag{4}$$

where W is the width of the flat panel and B denotes its thickness.

For this first group of analyses plate width, W, was chosen as 1000 mm, thickness, B, as 50 mm and yield strength of material, R_e, as 500 MPa.

The reference stress and limit load formulae employed in the assessment of the given flaw types can be found in Table 2.

Table 5 compares the loading conditions for flat plates. For through-thickness flaws, only the pure tension loading condition can be compared (R6 and FITNET do not contain solutions for pure bending or mixed tension and bending). For surface and embedded flaws, the assessment results under pure tension, pure bending and combined tension and bending loading conditions can be compared.

Besides loading conditions, how the tensile forces are applied (in other words, how the boundary conditions are defined) is another important factor that should be borne in mind before starting to compare individual assessment results obtained from BS7910:2005 and R6/FITNET for given flawed structures. For example, BS7910:2005 suggests two different formulae for both surface and embedded flaws; tensile forces can be applied via pin loading or fixed grip conditions. Conversely, in the R6/FITNET limit load solutions, the tensile forces are usually considered as pin jointed due to the nature of the numerical derivation methods.

The results of the flat plates containing through-thickness, surface and embedded flaws can be seen in Figures 10–17, where the results are introduced in terms of L_r and normalized geometrical factors.

A flat plate containing a through-thickness flaw subject to pure tension was assessed using BS formula (A.1) and R6/FITNET formulas (B.1) and (B.2). For the representation of assessment results flaw geometry was normalised with respect to the width of the plate and L_r was drawn for increasing *2a*/*W* values in Figure 10.

The next case considered was a flat plate containing a surface flaw. Differently from the through-thickness flaws, in the surface and embedded flaw cases, the dimensions of crack are expressed using two geometrical quantities; the length of the crack and the depth of the crack. When the length of the crack, *2c*, is normalised with respect to the width of the plate, *W*, and the depth of the crack, *a* or *2a*, is normalised with respect to the thickness of the plate, *B*, assessment surfaces as in Figure 11 are generated.

Turning back to the plate containing surface flaw, the assessment was conducted using BS formulae (A.2)-(A.3) and R6/FITNET formulae (B.3)-(B.6). In Figure 12, assessment results of the plate subject to pure tension can be found for increasing values of normalised crack length, *2c*/*W*, and the L_r vs. *a*/*B* curves represented in this diagram are two dimensional representations of the three dimensional assessment surfaces. In Figure 13 and Figure 14, the same geometrical case was considered under different loading conditions, pure bending and combined tension and bending respectively.

The last set of analyses was conducted for a flat plate containing an embedded flaw without off-set. The BS formula (A.4) and the R6/FITNET formulae (B.7)-(B.10) were utilised. The length of embedded flaw, *2c*, was normalised with respect to the length of the plate, *W*, and the depth of the embedded crack, *2a*, was normalized with respect to the thickness of plate, *W*. The results represented in Figure 15, Figure 16 and Figure 17 are of pure tension, pure bending and combined tension and bending loading conditions.

In these analyses, it was observed that:

1) BS7910:2005 and R6/FITNET (when plane stress Tresca, plane strain Tresca or plane stress Mises solutions, see equation (B.1) are used) arrive at identical results for through-thickness flaws, see Figure 10.

2) BS7910:2005 and R6/FITNET lead to different results for pure tension, pure bending and combined tension and bending loading conditions, when surface or embedded flaws are to be assessed.

3) The results for surface and embedded flaws differ significantly for deep flaws, where $a/B > 0.5$.

4) Drawing a conclusion on the change in the level of conservatism of an assessment made according to BS7910:2005 or R6/FITNET depends strongly on the load type. For example, when a plate containing a surface or an embedded flaw subject to pure tension loading condition is analysed, R6/FITNET delivers higher L_r values whereas BS7910:2005 arrives at higher L_r values for pure bending loading condition.

Since combined tension and bending loading conditions reflect actual experimental conditions, these should be compared with the results of an analysis and this type of analysis results should be regarded as basis for comparison of BS7910:2005 and R6/FITNET assessment results.

Another factor to be considered in the interpretation of results is that in R6/FITNET the solutions are classified as ‘global’ and ‘local’ limit load solutions. For through-thickness flaws, the reference stress solutions introduced in Annex A and the limit load solutions in Annex B are all global solutions (which can also be referred as net-section limit loads or reference stresses).

A ‘local’ limit load is the load needed to cause plasticity to spread across the remaining ligament and hence is expected to be more conservative than a global limit load. For the assessment of part-through flaws, BS7910:2005 reference stress solutions are based on the local yielding criteria, whereas R6/FITNET limit load solutions are available for both local and global yielding conditions. When the particular region of interest in a part-through flaw is known, the sensitivity of assessment can be changed accordingly in an R6/FITNET assessment. For example, if points near to the free surface are of interest, global solutions can be used.

ASSESSMENT OF PIPES / CYLINDERS CONTAINING AXIAL FLAWS

Pipes/cylinders containing axial flaws constitute the second group of assessments. For the examination of these structures, three different flaw types were considered, namely through-thickness, internal surface and external surface flaws, see also Figures 4–6 for the geometrical definitions of these flaws.

As in assessment of flat plates, membrane stress was calculated from the applied load (internal pressure in this case) using the following formulae:

$$P_m = \frac{p' r_i}{t} \quad \text{for thin-walled structures} \tag{5}$$

$$P_m = \frac{p'\left(r_o^2 + r_i^2\right)}{r_o^2 - r_i^2} \quad \text{for thick-walled structures} \tag{6}$$

where p' is internal pressure, r_o is outer radius, r_i is inner radius and t is the thickness of the pipe/cylinder.

The following assumptions were made for the definition of thin- and thick-walled pipe/cylinder:

$$\frac{r_i}{t} > 10 \quad \rightarrow \text{thin-walled}$$

$$\frac{r_i}{t} < 10 \quad \rightarrow \text{thick-walled}$$

Reference stress and limit load formulae employed in the assessment of the given flaw types can be found in Table 3.

Within this part of the study, pipes/cylinders with full-width, W, of 800 mm, inner radius, r_i, of 400 mm and thickness, t, of 20 mm (for thin-walled structures) or 200 mm (for thick-walled structures) were analysed. The material was assumed to have a yield strength of 600 MPa and the loading conditions considered were membrane and bending stresses of 150 MPa, and combined membrane/bending stress.

The loading conditions considered are summarized in Table 6.

It is possible to assess through-thickness flaws only in thin-walled pipes/cylinders using both BS7910:2005 and R6/FITNET. For this case, R6/FITNET limit load solution directly delivers a limit pressure value but BS7910:2005 requires calculation of internal pressure induced membrane stress (see equation (5)), and pressure induced through-wall bending can be ignored since through-wall bending stresses are not normally considered to contribute to the collapse load [7].

For the assessment of internal and external surface flaws, BS7910:2005 contains only one solution (equation (A.6)) and this formula is applicable to both thin- and thick-walled pipes/cylinders. If thick-walled pipes/cylinders are to be assessed, equation (A.6) should be used with caution and only within the application range, since it was originally derived for thin-walled structures.

In R6/FITNET, on the contrary, the internal surface flaw limit load solutions were derived specifically for thick-walled structures. R6/FITNET limit load solution for internal surface flaws also delivers a limit pressure value as in the assessment of through-thickness flaws and this limit pressure value can be used directly in the calculation of L_r. Membrane and through-wall bending stresses do not need to be calculated separately but this practical solution is a lower-bound solution that can lead to conservative values and can overestimate limit load [7].

For the assessment of external flaws, the structure of reference stress solutions in BS7910:2005 and limit load solutions in R6/FITNET are the same. Membrane and through-wall bending stresses are explicitly taken into the formulations. For this case, R6/FITNET limit load solutions have been derived/modified based on the flat plates containing surface flaws and it allows assessment of both thin- and thick-walled pipes/cylinders.

Finally, it should also be noted that BS7910:2005 advises the same formula for internal and external surface flaws, whereas R6/FITNET offers different sets of solutions for the two quite different cases.

Results of the pipes/cylinders containing through-thickness, internal surface and external surface flaws can be seen in Figures 18–27.

Assessment results of a pipe/cylinder containing a through-thickness flaw subject to internal pressure only can be seen in Figure 18. As for the plates, the length of the axial through-thickness flaw, *2c*, was normalized with respect to the length of the pipe/cylinder, *W*. The BS formula (A.5) and R6/FITNET formula (B.11) were used in this analysis.

For the next case analysed, a pipe/cylinder containing an internal surface flaw, BS formula (A.6) and R6/FITNET formulae (B.12)-(B.15) were employed. For pipes/cylinders, R6/FITNET allows consideration of defect-face pressures, besides global and local collapse conditions. For that very reason, there exist four different solutions for the assessment of a pipe/cylinder containing axial internal surface flaws. In Figure 19, first, four different R6/FITNET assessment results are presented. For the comparison of R6/FITNET results with the BS7910 results, one can see the following figure, Figure 20. Identical to the surface flaws in plates, the length of internal surface flaw, *2c*, was normalised with respect to the width of the pipe/cylinder, *W*, and the depth of the flaw, *a*, was normalised with respect to the thickness of pipe/cylinder, *B*. In both Figure 19 and Figure 20, the L_r vs. a/B results were drawn for increasing $2c/W$ values.

The last case considered for pipes/cylinders was external surface flaws. A thin-walled pipe/cylinder containing external surface flaw was analysed using BS formula (A.6) and R6/FITNET formulae (B.16)-(B.17). The results represented in Figure 21, Figure 22 and Figure 23 are of the pure tension, pure bending and combined tension and bending cases respectively.

Normalisation of flaw geometry is identical to the normalisation for internal surface flaws.

When a thick-walled pipe/cylinder containing an external surface flaw was analysed, it was seen that a limit load was not defined for all *a*/*B* and *2c*/*W* ratios, Figure 25. This is the consequence of the application range of these formulae affecting validity range of the analysis. When Figure 24 containing results of pure tension case is examined, it will be seen that L_r vs. *a*/*B* curves couldn't be drawn for all *2c*/*W* ratios for the geometry considered within the scope of this work. This is also apparent in Figure 26 and Figure 27, where the assessment results of pure bending and combined tension and bending loading cases can be found respectively.

In these analyses, it was observed that:

1) For through-thickness flaws, BS7910:2005 tends to give more conservative results. The multiplier of 1.2 introduced for the reference stress solution (A.5) to achieve a certain level of conservatism is partially responsible from this result. It should also be noted that in the original papers of Folias, this factor is not present.

2) In the internal surface flaw equations of BS7910:2005, the multiplier of 1.2 is incorporated into the formulations in order to assure certain level of conservatism as in through-thickness flaw case. When Figure 20 is observed, it can be seen that a BS7910:2005 assessment is slightly more conservative that an R6/FITNET assessment which was derived based on a lower bound estimation, that is already known to be overestimating limiting conditions approximately 5%.

3) As can be seen in Figures 19-20 and Annex B, in R6/FITNET four different limit load solutions are suggested. Solutions with defect-face pressure can be thought of a worst-case scenario where crack face contact in the compressive zone is ignored and leads to more conservative results.

4) Similar to the flat plates containing part-through flaws, the pipes/cylinders containing external surface flaws tend to show the same trend in conservatism in BS7910:2005 and R6/FITNET assessments. For a thin-walled pipe/cylinder subject to pure tension, R6/FITNET limit load solutions are more conservative whereas under pure bending loading conditions it is vice versa. Hence, in order to reach a conclusion, it is more appropriate to compare results of analyses conducted considering combined tension and bending loading condition; the difference in the assessment results seems to reduce, however R6/FITNET is still more conservative.

5) Plastic collapse assessments for thick-walled pipes/cylinders containing external surface flaws will be constrained by validity limits of limit load solutions (see B.2.3). For the case analysed in this study, limit loads were not defined for flaw depths greater than approximately half of the thickness of pipe, whereas BS7910:2005 reference stress solutions allow assessment of plastic collapse for a wider range of possible flaw shapes. In order to draw an ultimate conclusion on the conservatism or reliability of these analysis results, they should be compared with experimental results covering a wide range pipe/cylinder dimensions and flaw shapes and/or finite element results.

ASSESSMENT OF PIPES / CYLINDERS CONTAINING CIRCUMFERENTIAL FLAWS

Pipes/cylinders containing circumferential flaws constitute the third group of assessments. For the examination of these structures, identical to the analyses conducted for pipes/cylinders in the previous section, three different flaw types were considered, namely through-thickness, internal surface and external surface flaws, see also Figures 7–9 for the geometrical definitions of these flaws.

In addition to the internal pressure induced membrane stress conversions, equation (5)-(6), global bending stresses as functions of bending moments were calculated as follows:

$$P_{b,global} = \frac{6}{r_m t^2 A_b} M^b \tag{7}$$

where A_b is defined as follows for axisymmetric bend, $A_b = \left[\left(12-(t/r_m)^2\right)\Big/\left(2\left(6+(t/r_m)\right)\right)\right]$ for internal surface flaws and $A_b = \left[\left(12-(t/r_m)^2\right)\Big/\left(2\left(6-(t/r_m)\right)\right)\right]$ for external flaws. The global bending stresses calculated using equation (7) should be converted into part-through bending stresses, P_b, for use in conjunction with BS7910:2005 reference stress formulae. Furthermore, when pipes/cylinders containing circumferential flaws are loaded in tension, another membrane stress component will arise that can be calculated using the following formula for both thin- and thick-walled structures:

$$P_{m,axial} = \frac{F}{\pi\left(r_o^2 - r_i^2\right)} \tag{8}$$

Reference stress and limit load formulae utilised in the assessment of the given flaw types can be seen in Table 4.

Within this last part of the study, pipes/cylinders possessing inner radius, r_i, of 400 mm and thickness, t, of 20 mm made of a material with a yield strength of 600 MPa were analysed, for the loading condition where both membrane and bending stresses were chosen as 150 MPa.

Loading conditions considered can be found in Table 7. From this table, the limitation of reference stress solutions in the current version of BS7910 can be seen. Using the reference stress solutions of BS7910:2005, only thin-walled structures can be evaluated. However, if the limit load solutions in R6/FITNET are used, it is possible to assess both thin- and thick-walled structures. For comparison, within this study only the results of thin-walled structures will be discussed.

When the limit load equation systems for through-thickness flaws in Annex B are examined, it can be seen that the true position of the neutral axis can be calculated very accurately, if bending is to be taken into account. However, with the current equations in BS7910:2005 it is not possible to assess the location of the neutral axis and they can lead to unconservative results for the case where the flaw length exceeds 1/8 of circumference.

Usually, tension loads or global bending moments/stresses acting on structures are known. These values can be processed directly in an R6/FITNET assessment, whereas they should be converted into respective membrane and bending components in order to calculate reference stresses.

Results of the pipes/cylinders containing circumferentially oriented through-thickness and internal surface flaws can be seen in Figures 28–33.

For the analysis of through-thickness flaws oriented circumferentially, BS formula (A.7) and R6/FITNET formulae (B.20)-(B.21) were employed. The geometrical normalisation for circumferential flaws was quite different and required division of the half crack angle, θ, see Figure 7, by π. The assessment results of the pipe/cylinder containing a through-thickness flaw subject to pure tension, combined tension and bending, and in addition to combined tension and bending internal pressure as well can be found in Figure 28, Figure 29 and Figure 30 respectively.

During the analysis of the pipe/cylinder containing an internal surface flaw, BS formula (A.8) and R6/FITNET formulae (B.26)-(B.29) were utilized. In Figure 31, Figure 32 and Figure 33, the results of pure tension, combined tension and bending, and combined tension and bending with internal pressure cases are illustrated.

In these analyses, it was observed that in thin-walled pipes/cylinders:

1) For the case where through-thickness flaws are subject to pure tension, both BS7910:2005 and R6/FITNET deliver nearly the same results. The difference in the assessment of combined tension and bending case is marginal. However, when internal pressure is taken into account together with tension and bending, R6/FITNET results are more conservative than those of BS7910:2005.

2) During the analysis of internal surface flaws BS7910:2005 assessment results are more conservative for pure tension, combined tension and bending, and combined tension and bending together with internal pressure.

3) Since R6/FITNET contains limit load solutions derived based on global yielding for internal surface flaw case, BS7910:2005's local solutions are more conservative as expected.

Since both procedures advise the same formula for the assessment of internal and external surface flaws oriented circumferentially, a separate analysis for the external surface flaws was not conducted.

CONCLUSIONS AND OUTLOOK FOR BS 7910:2012

In the course of the revision of BS7910, intended to be published in 2012, a comprehensive comparative study was conducted in order to help the BS committee (WEE/37) decide whether the reference stress solutions of BS7910:2005 or the limit load solutions of R6/FITNET procedures should be used for the assessment of plastic collapse L_r.

These recent comparative studies have shown significant differences in the assessment of plastic collapse depending on whether the reference stress solutions in BS 7910:2005 or the limit load solutions in R6/FITNET are used for the calculation of L_r.

Hence, priority was given to the identification of the extent of this difference and to this end, two most-commonly tested and assessed geometries containing different flaw types were analysed. The results presented in the previous sections show merits and drawbacks of each assessment strategy for different conditions.

In the light of the results presented within this study, it is recommended to maintain the reference stress solutions for flat plates and pipes/cylinders containing axial flaws in the new BS7910. The reference stress solutions for pipes/cylinders containing circumferential flaws should be re-evaluated in order to enhance the assessment capability of the new BS7910.

Furthermore, the reference stress solutions recommended in BS7910 are mainly from local, and not global, limit load solutions, as they are more conservative. Ideally, for future code development, the same solutions should be specified and agreed upon in different standards and codes in order to reduce variability in analysing tests and making failure assessments. This way, material specific results and validations can be compared across industry and research papers.

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ANNEX A

REFERENCE STRESS SOLUTIONS

A.1 Reference stress solutions for flat plates

A.1.1 Flat plate with through-thickness flaw

$$\sigma_{ref} = \frac{P_b + \left(P_b^2 + 9P_m^2\right)^{0.5}}{3\left\{1 - \left(\frac{2a}{W}\right)\right\}} \qquad \text{(A.1)}$$

A.1.2 Flat plate with surface flaw

Normal bending restraint:

$$\sigma_{ref} = \frac{P_b + \left\{P_b^2 + 9P_m^2\left(1-\alpha''\right)^2\right\}^{0.5}}{3\left(1-\alpha''\right)^2} \qquad \text{(A.2)}$$

Negligible bending restraint

$$\sigma_{ref} = \frac{P_b + 3P_m\alpha'' + \left\{\left(P_b + 3P_m\alpha''\right)^2 + 9P_m^2\left(1-\alpha''\right)^2\right\}^{0.5}}{3\left(1-\alpha''\right)^2} \qquad \text{(A.3)}$$

where

$$\alpha'' = \left(a/B\right)/\left\{1+\left(B/c\right)\right\} \quad \text{for} \quad W \ge 2\left(c+B\right)$$

$$\alpha'' = \left(2a/B\right)\left(c/W\right) \quad \text{for} \quad W < 2\left(c+B\right)$$

A.1.3 Flat plate with embedded flaw

$$\sigma_{ref} = \frac{P_b + 3P_m\alpha'' + \left[\left(P_b + 3P_m\alpha''\right)^2 + 9P_m^2\left\{\left(1-\alpha''\right)^2 + 4\left(\frac{p\alpha''}{B}\right)\right\}\right]^{0.5}}{3\left\{\left(1-\alpha''\right)^2 + 4\left(\frac{p\alpha''}{B}\right)\right\}} \qquad \text{(A.4)}$$

where

$$\alpha'' = \left(2a/B\right)/\left\{1+\left(B/c\right)\right\} \quad \text{for} \quad W \ge 2\left(c+B\right)$$

$$\alpha'' = \left(4a/B\right)\left(c/W\right) \quad \text{for} \quad W < 2\left(c+B\right)$$

A.2 Reference stress solutions for pipes/cylinders with axial flaws

A.2.1 Through-thickness flaw in pipe/cylinder oriented axially

$$\sigma_{ref} = 1.2M_T P_m + \frac{2P_b}{3\left(1-\frac{2a}{W}\right)} \qquad \text{(A.5)}$$

where

$$M_T = \left\{1 + 1.6\left(a^2/r_i t\right)\right\}^{0.5}$$

A.2.2 Internal surface flaw in cylinder oriented axially

$$\sigma_{ref} = 1.2M_s P_m + \frac{2P_b}{3\left(1-\alpha''\right)^2} \qquad \text{(A.6)}$$

where

$$M_s = \frac{1-\left\{a/\left(tM_T\right)\right\}}{1-\left(a/t\right)}$$

$$M_T = \sqrt{1 + 1.6\left(\frac{c^2}{r_i t}\right)}$$

$$\alpha'' = \left(\frac{a}{t}\right)\Big/\left(1+\frac{t}{c}\right) \quad \text{for} \quad W \ge 2\left(c+t\right)$$

$$\alpha'' = 2\left(\frac{a}{t}\right)\left(\frac{c}{\pi r_i}\right) \quad \text{for} \quad W < 2\left(c+t\right)$$

A.2.3 External surface flaw in cylinder oriented axially

The reference stress is calculated from (A.6).

A.3 Reference stress solutions for pipes/cylinders with circumferential flaws

A.3.1 Through-thickness flaw in cylinder oriented circumferentially

$$\sigma_{ref} = \frac{\pi\left(P_{m,a} + P_{m,p}\right)}{\pi - \frac{a}{r_i} - 2\arcsin\left(\frac{1}{2}\sin\frac{a}{r_i}\right)} + \frac{\pi P_{m,b}\left(r_o^4 - r_i^4\right)}{\left\{\pi - \frac{a}{r_i} - 2\frac{\sin^2\left(\frac{a}{r_i}\right)}{\pi - \frac{a}{r_i}} - \frac{\sin\left(\frac{2a}{r_i}\right)}{2}\right\}\left(4r_o r^2 B\right)} + \frac{2P_b}{3\left(1-\frac{2a}{\pi r_i}\right)} \qquad \text{(A.7)}$$

A.3.2 Internal surface flaw in cylinder oriented circumferentially

$$\sigma_{ref} = \frac{P_m\left\{\pi\left(1-\frac{a}{t}\right) + 2\left(\frac{a}{t}\right)\sin\left(\frac{c}{r}\right)\right\}}{\left(1-\frac{a}{t}\right)\left\{\pi - \left(\frac{c}{r}\right)\left(\frac{a}{t}\right)\right\}} + \frac{2P_b}{3\left(1-\alpha''\right)^2} \qquad \text{(A.8)}$$

where

$$\alpha'' = \frac{\frac{a}{t}}{\left\{1+\left(\frac{t}{c}\right)\right\}} \quad \text{for} \quad \pi r \ge c+t$$

$$\alpha'' = \left(\frac{a}{t}\right)\left(\frac{\mathrm{c}}{\pi \mathrm{r}}\right) \quad \text{for} \quad \pi r < c+t$$

A.3.3 External surface flaw in cylinder oriented circumferentially

The reference stress is calculated from (A.8).

ANNEX B

LIMIT LOAD SOLUTIONS

B.1 Limit load solutions for flat plates

B.1.1 Flat plate with through-thickness flaw

$$L_r^N = \frac{F_e^N}{WBR_e}, \quad \beta = \frac{2a}{W}, \quad \gamma = \frac{2}{\sqrt{3}}$$

Plane stress Tresca, plane stress Mises and plane strain Tresca solutions:

$$L_r^N = 1-\beta \quad \text{for} \quad 0 \le \beta < 1 \tag{B.1}$$

Plane strain Mises solution:

$$L_r^N = \gamma(1-\beta) \quad \text{for} \quad 0 \le \beta < 1 \tag{B.2}$$

B.1.2 Flat plate with surface flaw

$$L_r^N = \frac{F_e^N}{WBR_e}, \quad L_r^b = \frac{4M_e^b}{WB^2R_e}, \quad \lambda = \frac{M^b}{BF^N} = \frac{1}{6}\frac{\sigma_b}{\sigma_m},$$

$$\alpha = \frac{a}{B}, \quad \beta = \frac{2c}{W}, \quad \psi = \frac{c}{B}$$

Global solution

$$L_r^N = \begin{cases} \dfrac{d_1}{2\lambda+\alpha\beta+\sqrt{(2\lambda+\alpha\beta)^2+d_1}} & \text{for} \quad \alpha \le \alpha_0 \\ \dfrac{d_2}{2\lambda+\beta\dfrac{1-\alpha}{1-\beta}+\sqrt{\left(2\lambda+\beta\dfrac{1-\alpha}{1-\beta}\right)^2+\dfrac{d_2}{1-\beta}}} & \text{for} \quad \alpha > \alpha_0 \end{cases} \tag{B.3}$$

$$L_r^b = \begin{cases} \dfrac{4\lambda d_1}{2\lambda+\alpha\beta+\sqrt{(2\lambda+\alpha\beta)^2+d_1}} & \text{for} \quad \alpha \le \alpha_0 \\ \dfrac{4\lambda d_2}{2\lambda+\beta\dfrac{1-\alpha}{1-\beta}+\sqrt{\left(2\lambda+\beta\dfrac{1-\alpha}{1-\beta}\right)^2+\dfrac{d_2}{1-\beta}}} & \text{for} \quad \alpha > \alpha_0 \end{cases} \tag{B.4}$$

where

$$d_1 = (1-\alpha\beta)^2 + 2\alpha^2\beta(1-\beta)$$

$$d_2 = (1-\alpha\beta)\left[2-\left(\frac{1-\alpha\beta}{1-\beta}\right)\right]+2\alpha\beta(1-\alpha)$$

$$\alpha_0 = -\left(\lambda-\frac{1}{2}\right)+\sqrt{\left(\lambda-\frac{1}{2}\right)^2+\frac{\lambda}{1-\frac{1}{2}\beta}}$$

Local solutions

$$L_r^N = \begin{cases} \dfrac{d_1}{2\lambda+\dfrac{\alpha\psi}{1+\psi}+\sqrt{\left(2\lambda+\dfrac{\alpha\psi}{1+\psi}\right)^2+d_1}} & \text{for} \quad \alpha \le \alpha_0 \\ \dfrac{d_2}{2\lambda+\psi(1-\alpha)+\sqrt{(2\lambda+\psi(1-\alpha))^2+(1+\psi)d_2}} & \text{for} \quad \alpha > \alpha_0 \end{cases} \tag{B.5}$$

$$L_r^b = \begin{cases} \dfrac{4\lambda d_1}{2\lambda+\dfrac{\alpha\psi}{1+\psi}+\sqrt{\left(2\lambda+\dfrac{\alpha\psi}{1+\psi}\right)^2+d_1}} & \text{for} \quad \alpha \le \alpha_0 \\ \dfrac{4\lambda d_2}{2\lambda+\psi(1-\alpha)+\sqrt{(2\lambda+\psi(1-\alpha))^2+(1+\psi)d_2}} & \text{for} \quad \alpha > \alpha_0 \end{cases} \tag{B.6}$$

where

$$d_1 = \left(1-\frac{\alpha\psi}{1+\psi}\right)^2+\frac{2\alpha^2\psi}{(1+\psi)^2}$$

$$d_2 = \left(1-\frac{\alpha\psi}{1+\psi}\right)[1-\psi(1-\alpha)]+\frac{2\alpha(1-\alpha)\psi}{1+\psi}$$

$$\alpha_0 = -\left(\lambda-\frac{1}{2}\right)+\sqrt{\left(\lambda-\frac{1}{2}\right)^2+\frac{2(1+\psi)\lambda}{2+\psi}}$$

For local case the solutions are limited to

$$\beta < \frac{\psi}{1+\psi}$$

B.1.3 Flat plate with embedded flaw

$$L_r^N = \frac{F_e^N}{WBR_e}, \quad L_r^b = \frac{4M_e^b}{WB^2R_e}, \quad \lambda = \frac{M^b}{BF^N} = \frac{1}{6}\frac{\sigma_b}{\sigma_m}, \quad \alpha = \frac{a}{B},$$

$$\beta = \frac{2c}{W}, \quad k = \frac{\frac{B}{2}-p-a}{B}, \quad \psi = \frac{c}{B}$$

$$L_r^N = \begin{cases} \dfrac{c_1}{2(\lambda+\alpha\beta)+\sqrt{4(\lambda+\alpha\beta)^2+c_1}} & \text{for} \quad \alpha \le \min(\alpha_1, \alpha_2) \\ \dfrac{c_2}{2[(1-\beta)\lambda+\beta k]+\sqrt{4[(1-\beta)\lambda+\beta k]^2+c_2}} & \text{for} \quad \alpha_1 < \alpha \le \alpha_2 \end{cases} \tag{B.7}$$

$$L_r^b = \begin{cases} \dfrac{4\lambda c_1}{2(\lambda+\alpha\beta)+\sqrt{4(\lambda+\alpha\beta)^2+c_1}} & \text{for} \quad \alpha \le \min(\alpha_1, \alpha_2) \\ \dfrac{4\lambda c_2}{2[(1-\beta)\lambda+\beta k]+\sqrt{4[(1-\beta)\lambda+\beta k]^2+c_2}} & \text{for} \quad \alpha_1 < \alpha \le \alpha_2 \end{cases} \tag{B.8}$$

where

$$c_1 = 1-8\alpha\beta k-4(\alpha\beta)^2$$

$$c_2 = (1-\beta)\left(1-\frac{4\beta k^2}{1-\beta}-4\beta\alpha^2\right)$$

$$\alpha_1 = (k-\lambda)(1-\beta)+\sqrt{(k-\lambda)^2(1-\beta)^2+\left(\frac{1}{4}-k^2+2k\lambda\right)}$$

$$\alpha_1 = \begin{cases} k(1-\beta)+\sqrt{\frac{1}{4}-k^2\beta(2-\beta)} & \text{for pure tension } \lambda=0 \\ \frac{k}{1-\beta} & \text{for pure bending } \lambda\to\infty \end{cases}$$

$$\alpha_2 = \frac{1}{2}-k$$

Local solutions ($d = B + c$ and $t_1 = B$):

$$L_r^N = \begin{cases} \dfrac{c_1}{2\left(\lambda+\frac{\alpha\psi}{1+\psi}\right)+\sqrt{4\left(\lambda+\frac{\alpha\psi}{1+\psi}\right)^2+c_1}} & \text{for } \alpha \le \min(\alpha_1,\alpha_2) \\ \dfrac{c_2}{\frac{2(\lambda+\psi k)}{1+\psi}+\sqrt{4\left(\frac{\lambda+\psi k}{1+\psi}\right)^2+c_2}} & \text{for } \alpha_1 < \alpha \le \alpha_2 \end{cases} \tag{B.9}$$

$$L_r^b = \begin{cases} \dfrac{4\lambda c_1}{2\left(\lambda+\frac{\alpha\psi}{1+\psi}\right)+\sqrt{4\left(\lambda+\frac{\alpha\psi}{1+\psi}\right)^2+c_1}} & \text{for } \alpha \le \min(\alpha_1,\alpha_2) \\ \dfrac{4\lambda c_2}{\frac{2(\lambda+\psi k)}{1+\psi}+\sqrt{4\left(\frac{\lambda+\psi k}{1+\psi}\right)^2+c_2}} & \text{for } \alpha_1 < \alpha \le \alpha_2 \end{cases} \tag{B.10}$$

where

$$c_1 = 1-\frac{8\alpha k\psi}{1+\psi}-4\left(\frac{\alpha\psi}{1+\psi}\right)^2$$

$$c_2 = \frac{1}{1+\psi}\left(1-4\psi k^2-\frac{4\psi\alpha^2}{1+\psi}\right)$$

$$\alpha_1 = \frac{k-\lambda}{1+\psi}+\sqrt{\left(\frac{k-\lambda}{1+\psi}\right)^2+\left(\frac{1}{4}-k^2+2k\lambda\right)}$$

$$\alpha_1 = \begin{cases} \frac{k}{1+\psi}+\sqrt{\frac{1}{4}-\frac{k^2\psi(2+\psi)}{(1+\psi)^2}} & \text{for pure tension } \lambda=0 \\ k(1+\psi) & \text{for pure bending } \lambda\to\infty \end{cases}$$

$$\alpha_2 = \frac{1}{2}-k$$

Global solutions are a net-section collapse solution valid for

$$\frac{1}{2} > k \ge 0 \text{ and } \alpha \le \frac{1}{2}-k$$

Local solutions are valid for

$$\frac{1}{2} > k \ge 0\,,\ \alpha \le \frac{1}{2}-k \text{ and } \beta < \frac{\psi}{1+\psi}$$

B.2 Limit load solutions for pipes/cylinders with axial flaws

B.2.1 Through-thickness cracks in pipe/cylinder oriented axially

$$\eta = \frac{t}{r_m}\,,\quad \phi = \frac{t}{a}$$

$$\frac{P_e}{R_e} = \frac{\eta}{\sqrt{1+1.05\frac{\eta}{\phi^2}}} \tag{B.11}$$

B.2.2 Internal surface flaw in pipe/cylinder oriented axially

$$\alpha = \frac{a}{t}\,,\quad \eta = \frac{t}{r_m}\,,\quad \phi = \frac{a}{c}$$

(a) Global solutions:

(i) Without defect-face pressure:

$$\frac{P_e}{R_e} = \frac{\alpha\eta}{\left(1-\frac{1}{2}\eta\right)M_g}+\ln\left(\frac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\right) \tag{B.12}$$

where

$$M_g = \sqrt{1+1.05\frac{\alpha\eta}{\phi^2\left(1-\frac{1}{2}\eta\right)}}$$

(ii) With defect-face pressure:

$$\frac{P_e}{R_e} = \frac{\alpha\eta}{\left(1-\frac{1}{2}\eta\right)M_g}+\frac{1-\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\ln\left(\frac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\right) \tag{B.13}$$

(b) Local solutions:

(i) Without defect-face pressure ($d = c + s_1(1-\alpha)$ and $t_1 = B$):

$$\frac{P_e}{R_e} = \frac{s_1(1-\alpha)\ln\left(\frac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta}\right)+c\ln\left(\frac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\right)}{c+s_1(1-\alpha)} \tag{B.14}$$

where

$$s_1 = \frac{\alpha\eta c}{\left(1-\frac{1}{2}\eta\right)M_g\ln\left(1+\frac{\alpha\eta}{1-\frac{1}{2}\eta}\right)-\alpha\eta}$$

(ii) With defect-face pressure ($d = c + s_2(1-\alpha)$ and $t_1 = B$):

$$\frac{P_e}{R_e} = \frac{s_2(1-\alpha)\ln\left(\frac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta}\right)+c\frac{1-\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\ln\left(\frac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\right)}{c+s_2(1-\alpha)} \tag{B.15}$$

where

$$s_2 = \frac{\alpha\eta c}{\left(1-\frac{1}{2}\eta\right)M_g\left[\ln\left(\frac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta}\right)-\frac{1-\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\ln\left(\frac{1+\frac{1}{2}\eta}{1-\frac{1}{2}\eta+\alpha\eta}\right)\right]-\alpha\eta}$$

B.2.3 External surface flaw in pipe/cylinder oriented axially

$$L_r^N = \frac{\sigma_{n,m}}{R_e}, \quad L_r^b = \frac{2}{3}\frac{\sigma_{n,b}}{R_e}, \quad \lambda = \frac{1}{6}\frac{\sigma_b}{\sigma_m}, \quad \alpha = \frac{a}{t}, \quad \psi = \frac{c}{t},$$

$$\beta = \frac{2c}{W}$$

Local solutions ($W' = t + c$ and $t' = t$):

$$L_r^N = \begin{cases} \dfrac{d_1}{2\lambda + \dfrac{\alpha\psi}{1+\psi} + \sqrt{\left(2\lambda + \dfrac{\alpha\psi}{1+\psi}\right)^2 + d_1}} & \text{for } \alpha \le \alpha_0 \\ \dfrac{d_2}{2\lambda + \psi(1-\alpha) + \sqrt{\left(2\lambda + \psi(1-\alpha)\right)^2 + (1+\psi)d_2}} & \text{for } \alpha > \alpha_0 \end{cases} \tag{B.16}$$

$$L_r^b = \begin{cases} \dfrac{4\lambda d_1}{2\lambda + \dfrac{\alpha\psi}{1+\psi} + \sqrt{\left(2\lambda + \dfrac{\alpha\psi}{1+\psi}\right)^2 + d_1}} & \text{for } \alpha \le \alpha_0 \\ \dfrac{4\lambda d_2}{2\lambda + \psi(1-\alpha) + \sqrt{\left(2\lambda + \psi(1-\alpha)\right)^2 + (1+\psi)d_2}} & \text{for } \alpha > \alpha_0 \end{cases} \tag{B.17}$$

where

$$d_1 = \left(1 - \frac{\alpha\psi}{1+\psi}\right)^2 + \frac{2\alpha^2\psi}{(1+\psi)^2}$$

$$d_2 = \left(1 - \frac{\alpha\psi}{1+\psi}\right)\left[1 - \psi(1-\alpha)\right] + \frac{2\alpha(1-\alpha)\psi}{1+\psi}$$

$$\alpha_0 = -\left(\lambda - \frac{1}{2}\right) + \sqrt{\left(\lambda - \frac{1}{2}\right)^2 + \frac{2(1+\psi)\lambda}{2+\psi}}$$

For pure tension ($\lambda = 0$) and pure bending ($\lambda \to \infty$)

$$\alpha_0 = \begin{cases} 1 & \text{for } \lambda = 0 \\ \dfrac{\psi + 1}{\psi + 2} & \text{for } \lambda \to \infty \end{cases}$$

The solutions are limited to

$$\beta \le \frac{\psi}{1+\psi}$$

B.3 Limit load solutions for pipes/cylinders with circumferential flaws

$$L_r^N = \frac{F_e^N}{2\pi r_m t R_e}, \quad L_r^b = \frac{M_e^b}{4 r_m^2 t R_e}, \quad \eta = \frac{t}{r_m}$$

$$\lambda = \frac{M^b}{r_m F^N} = \frac{L_r^b}{\frac{\pi}{2} L_r^N} \quad \text{for thick-walled pipe/cylinder}$$

$$\lambda = \frac{M^b}{r_m\left(F^N + \pi r_m^2 p'\right)} = \frac{L_r^b}{\frac{\pi}{2} L_r^{pN}} \quad \text{for thin-walled pipe/cylinder}$$

B.3.1 Through-thickness flaw in pipe/cylinder oriented circumferentially

$$\alpha = 1, \quad \theta = \frac{a}{r_m}$$

a) Thick-walled cylinders under combined tension and bending

Whole crack inside the tensile stress zone ($\theta + \beta \le \pi$):

$$\frac{\beta}{\pi} = \frac{1}{2}\left(1 - \frac{\theta}{\pi} - L_r^N\right) \tag{B.18}$$

$$L_r^b = f_b(\eta)\sin\beta - \frac{1}{2} f_c(\eta)\sin\theta \tag{B.19}$$

where

$$f_b = 1 + \frac{1}{12}\eta^2$$

$$f_c = 1 + \frac{1}{6}\eta^2$$

b) Thin-walled cylinders under combined tension and bending with internal pressure:

$$L_r^p = \frac{\left(r_m - \frac{t}{2}\right)^2 P_e}{2 r_m t R_e} \approx \frac{r_m P_e}{2tR_e}$$

$$\chi = \frac{F^N}{\pi r_m^2 p'} = \frac{L_r^N}{L_r^p}, \quad L_r^{pN} = L_r^p + L_r^N = (1+\chi) L_r^p$$

$$S_{a1} = \frac{1}{2}\left(\frac{2L_r^{pN}}{1+\chi} + \sqrt{4 - 3\left(\frac{2L_r^{pN}}{1+\chi}\right)^2}\right)$$

$$S_{a2} = \frac{1}{2}\left(\frac{2L_r^{pN}}{1+\chi} - \sqrt{4 - 3\left(\frac{2L_r^{pN}}{1+\chi}\right)^2}\right)$$

Global solutions:

Whole crack inside the tensile stress zone ($\theta + \beta \le \pi$)

$$\frac{\beta}{\pi} = \frac{S_{a1}}{S_{a1} - S_{a2}}\left(1 - \frac{\theta}{\pi} - \frac{L_r^{pN}}{S_{a1}}\right) \tag{B.20}$$

$$L_r^b = \frac{1}{2}\left[(S_{a1} - S_{a2})\sin\beta - S_{a1}\sin\theta\right] \tag{B.21}$$

B.3.2 Internal surface flaw in pipe/cylinder oriented circumferentially

$$\alpha = \frac{a}{t}, \quad \theta = \frac{c}{r_m - \frac{t}{2}}$$

a) Thick-walled cylinders under combined tension and bending:

(i) Whole crack inside the tensile stress zone ($\theta + \beta \le \pi$):

$$\frac{\beta}{\pi} = \frac{1}{2}\left(1 - f_a(\eta,\alpha)\,\alpha\frac{\theta}{\pi} - L_r^N\right) \tag{B.22}$$

$$L_r^b = f_b(\eta)\sin\beta - \frac{1}{2}\alpha f_c(\eta,\alpha)\sin\theta \quad (B.23)$$

(ii) Part of the crack inside the compression zone ($\theta+\beta>\pi$):

$$\frac{\beta}{\pi} = 1 - \frac{1+L_r^N - \left[1-f_e(\eta,\alpha)\right]\frac{\theta}{\pi}}{2f_e(\eta,\alpha)} \quad (B.24)$$

$$L_r^b = f_b(\eta)\left[f_d(\eta,\alpha)\sin\beta + \frac{1}{2}\left(1-f_d(\eta,\alpha)\right)\sin\theta\right] \quad (B.25)$$

where

$$f_a = 1-\frac{1}{2}\eta+\frac{1}{2}\alpha\eta$$

$$f_b = 1+\frac{1}{12}\eta^2$$

$$f_c = 1-\eta+\frac{1}{4}\eta^2+\alpha\eta-\frac{1}{2}\alpha\eta^2+\frac{1}{3}\alpha^2\eta^2$$

$$f_d = (1-\alpha)\left[1+\alpha\eta-\frac{1}{6}\alpha\eta^2+\frac{1}{3}\alpha^2\eta^2+\frac{1}{12}\eta^2\right]\Big/ f_b(\eta)$$

$$f_e = 1-\alpha+\frac{1}{2}\alpha\eta-\frac{1}{2}\alpha^2\eta$$

b) Thin-walled cylinders under combined tension and bending with internal pressure:

$$L_r^p = \frac{\left(r_m-\frac{t}{2}\right)^2 P_e}{2r_m t R_e} \approx \frac{r_m P_e}{2tR_e}, \quad \chi = \frac{F^N}{\pi\cdot r_m^2 p'} = \frac{L_r^N}{L_r^p}$$

$$L_r^{pN} = L_r^p + L_r^N = (1+\chi)L_r^p$$

$$S_{a1} = \frac{1}{2}\left(\frac{2L_r^{pN}}{1+\chi} + \sqrt{4-3\left(\frac{2L_r^{pN}}{1+\chi}\right)^2}\right)$$

$$S_{a2} = \frac{1}{2}\left(\frac{2L_r^{pN}}{1+\chi} - \sqrt{4-3\left(\frac{2L_r^{pN}}{1+\chi}\right)^2}\right)$$

Global solutions:

(i) Whole crack inside the tensile stress zone ($\theta+\beta\le\pi$)

$$\frac{\beta}{\pi} = \frac{S_{a1}}{S_{a1}-S_{a2}}\left(1-\alpha\frac{\theta}{\pi}-\frac{L_r^{pN}}{S_{a1}}\right) \quad (B.26)$$

$$L_r^b = \frac{1}{2}\left[(S_{a1}-S_{a2})\sin\beta - S_{a1}\alpha\sin\theta\right] \quad (B.27)$$

(ii) Part of the crack inside the compression zone ($\theta+\beta>\pi$)

$$\frac{\beta}{\pi} = \frac{1}{(S_{a1}-S_{a2})(1-\alpha)}\left(S_{a1}-(S_{a1}-S_{a2})\alpha - S_{a2}\,\alpha\frac{\theta}{\pi} - L_r^{pN}\right) \quad (B.28)$$

$$L_r^b = \frac{1}{2}\left[(S_{a1}-S_{a2})(1-\alpha)\sin\beta - S_{a2}\alpha\sin\theta\right] \quad (B.29)$$

B.3.3 External surface flaw in pipe/cylinder oriented circumferentially

$$\alpha = \frac{a}{t}, \quad \theta = \pi$$

a) Thick-walled cylinders under combined tension and bending:

$$\frac{\beta}{\pi} = 1 - \frac{1+L_r^N - \left[1-f_e(\eta,\alpha)\right]}{2f_e(\eta,\alpha)} \quad (B.30)$$

$$L_r^b = f_b(\eta)\left[f_d(\eta,\alpha)\sin\beta\ \right] \quad (B.31)$$

where

$$f_a = 1-\frac{1}{2}\eta+\frac{1}{2}\alpha\eta$$

$$f_b = 1+\frac{1}{12}\eta^2$$

$$f_c = 1-\eta+\frac{1}{4}\eta^2+\alpha\eta-\frac{1}{2}\alpha\eta^2+\frac{1}{3}\alpha^2\eta^2$$

$$f_d = (1-\alpha)\left[1+\alpha\eta-\frac{1}{6}\alpha\eta^2+\frac{1}{3}\alpha^2\eta^2+\frac{1}{12}\eta^2\right]\Big/ f_b(\eta)$$

$$f_e = 1-\alpha+\frac{1}{2}\alpha\eta-\frac{1}{2}\alpha^2\eta$$

b) Thin-walled cylinders under combined tension and bending with internal pressure:

$$\alpha = \frac{a}{t}, \quad \theta = \pi, \quad L_r^p = \frac{\left(r_m-\frac{t}{2}\right)^2 P_e}{2r_m t R_e} \approx \frac{r_m P_e}{2tR_e}$$

$$\chi = \frac{F^N}{\pi\cdot r_m^2 p'} = \frac{L_r^N}{L_r^p}, \quad L_r^{pN} = L_r^p + L_r^N = (1+\chi)L_r^p$$

$$S_{a1} = \frac{1}{2}\left(\frac{2L_r^{pN}}{1+\chi} + \sqrt{4-3\left(\frac{2L_r^{pN}}{1+\chi}\right)^2}\right)$$

$$S_{a2} = \frac{1}{2}\left(\frac{2L_r^{pN}}{1+\chi} - \sqrt{4-3\left(\frac{2L_r^{pN}}{1+\chi}\right)^2}\right)$$

Global solutions:

$$\frac{\beta}{\pi} = \frac{1}{(S_{a1}-S_{a2})(1-\alpha)}\left(S_{a1}-(S_{a1}-S_{a2})\alpha - S_{a2}\,\alpha - L_r^{pN}\right) \quad (B.32)$$

$$L_r^b = \frac{1}{2}\left[(S_{a1}-S_{a2})(1-\alpha)\sin\beta\ \right] \quad (B.33)$$

Structure	Flaw type	Orientation of flaw
Flat plate	through-thickness	-
	surface	-
	embedded	-
Pipe / Cylinder	through-thickness	axial
	internal surface	axial
	external surface	axial
Pipe / Cylinder	through-thickness	circumferential
	internal surface	circumferential
	external surface	circumferential

TABLE 1: OVERVIEW OF ANALYTICAL WORK

Flaw type	BS7910:2005 Reference stress solution	FITNET / R6 Limit load solution
through-thickness	(A.1)	(B.1) – (B.2)
surface	(A.2) – (A.3)	(B.3) – (B.6)
embedded	(A.4)	(B.7) – (B.10)

TABLE 2: FORMULAE USED TO ASSESS PLASTIC COLLAPSE OF FLAT PLATES

Flaw type	BS7910:2005 Reference stress solution	FITNET / R6 Limit load solution
through-thickness	(A.5)	(B.11)
internal surface	(A.6)	(B.12) – (B.15)
external surface	(A.6)	(B.16) – (B.17)

TABLE 3: FORMULAE USED TO ASSESS PLASTIC COLLAPSE OF PIPES/CYLINDERS WITH AXIAL FLAWS

Flaw type	BS7910:2005 Reference stress solution	FITNET / R6 Limit load solution
through-thickness	(A.7)	(B.18) – (B.19) (B.20) – (B.21)
internal surface	(A.8)	(B.22) – (B.25) (B.26) – (B.29)
external surface	(A.8)	(B.30) – (B.31) (B.32) – (B.33)

TABLE 4: FORMULAE USED TO ASSESS PLASTIC COLLAPSE OF PIPES/CYLINDERS WITH CIRCUMFERENTIAL FLAWS

	Pure Tension		Pure Bending		Combined Tension and Bending	
	FITNET / R6	BS 7910:2005	FITNET / R6	BS 7910:2005	FITNET / R6	BS 7910:2005
Through-thickness flaw	√	√	-	√	-	√
Surface flaw	√ (‡)	√ (‡, §)	√	√	√ (‡)	√ (‡, §)
Embedded flaw	√ (‡)	√	√	√	√ (‡)	√

Application of tension: ‡ Pin loading, § Fixed grip

TABLE 5: LOADING CONDITIONS CONSIDERED IN THE ANALYSES OF FLAT PLATES

	Membrane (Hoop) Stress		Bending Stress		Combined Membrane and Bending Stresses	
	FITNET / R6	BS 7910:2005	FITNET / R6	BS 7910:2005	FITNET / R6	BS 7910:2005
Through-thickness flaw ‡	√*	√	-	√	-	√
Internal surface flaw	√*§	√	-	√	-	√
External surface flaw	√	√	√	√	√	√

* lower-bound solution(s) available
‡ solutions in both procedures available only for thin-walled pipes/cylinders
§ solutions available only for thick-walled pipes/cylinders

TABLE 6: LOADING CONDITIONS CONSIDERED IN THE ANALYSES OF PIPES WITH AXIAL FLAWS

	Membrane Stress		Bending Stress		Combined Membrane and Bending Stresses	
	FITNET / R6	BS 7910:2005	FITNET / R6	BS 7910:2005	FITNET / R6	BS 7910:2005
Through-thickness flaw thick-walled	√*	-	√	-	√	-
Through-thickness flaw thin-walled with internal pressure	√*	√	√	√	√	√
Internal surface flaw thick-walled	√	-	√	-	√	-
Internal surface flaw thin-walled with internal pressure	√	√	√	√	√	√
External surface flaw thick-walled	√	-	√	-	√	-
External surface flaw thin-walled with internal pressure	√	√	√	√	√	√

TABLE 7: LOADING CONDITIONS CONSIDERED IN THE ANALYSES OF PIPES WITH CIRCUMFERENTIAL FLAWS

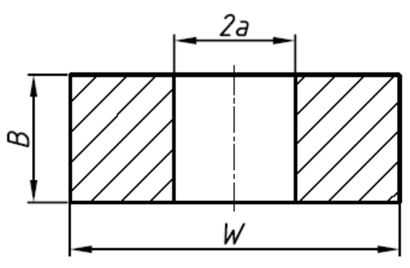

FIGURE 1: CROSS-SECTION OF A FLAT PANEL WITH THROUGH-THICKNESS FLAW [4]

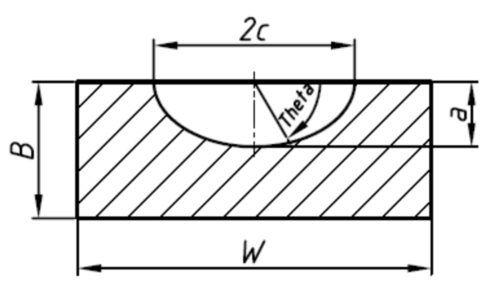

FIGURE 2: CROSS-SECTION OF A FLAT PANEL WITH SURFACE FLAW [4]

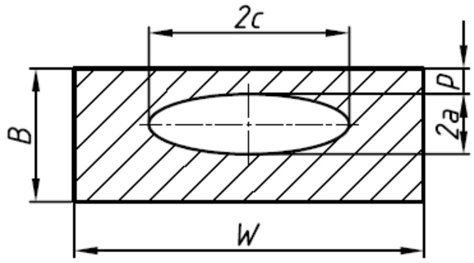

FIGURE 3: CROSS-SECTION OF A FLAT PANEL WITH EMBEDDED FLAW [4]

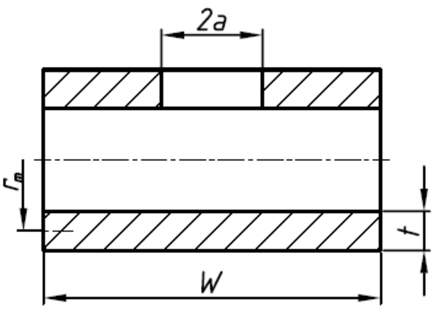

FIGURE 4: PIPE/CYLINDER WITH AXIAL THROUGH-THICKNESS FLAW [4]

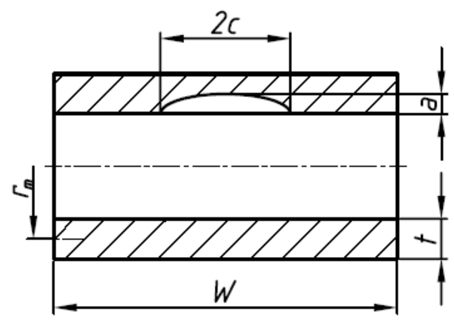

FIGURE 5: PIPE/CYLINDER WITH AXIAL INTERNAL SURFACE FLAW [4]

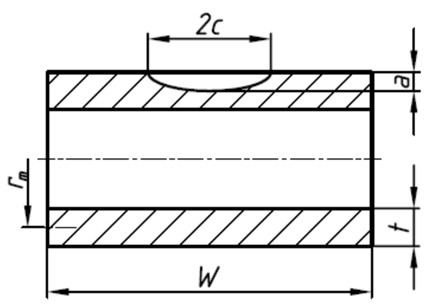

FIGURE 6: PIPE/CYLINDER WITH AXIAL EXTERNAL SURFACE FLAW [4]

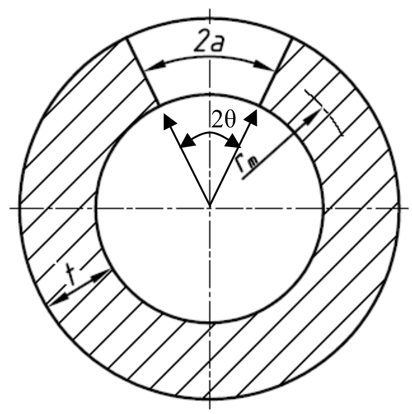

FIGURE 7: PIPE/CYLINDER WITH CIRCUMFERENTIAL THROUGH-THICKNESS FLAW [4]

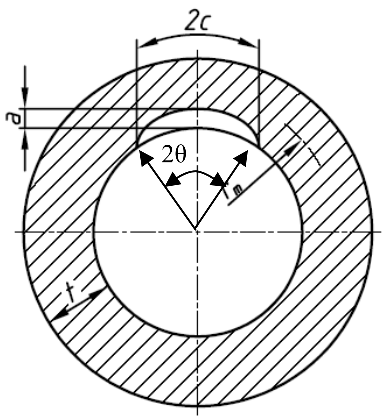

FIGURE 8: PIPE/CYLINDER WITH CIRCUMFERENTIAL INTERNAL SURFACE FLAW [4]

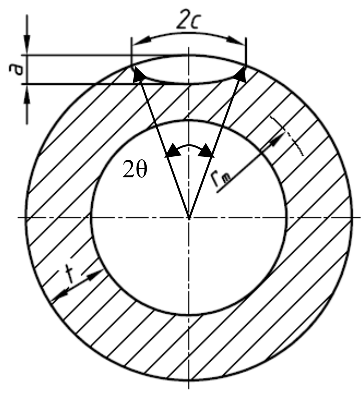

FIGURE 9: PIPE/CYLINDER WITH CIRCUMFERENTIAL EXTERNAL SURFACE FLAW [4]

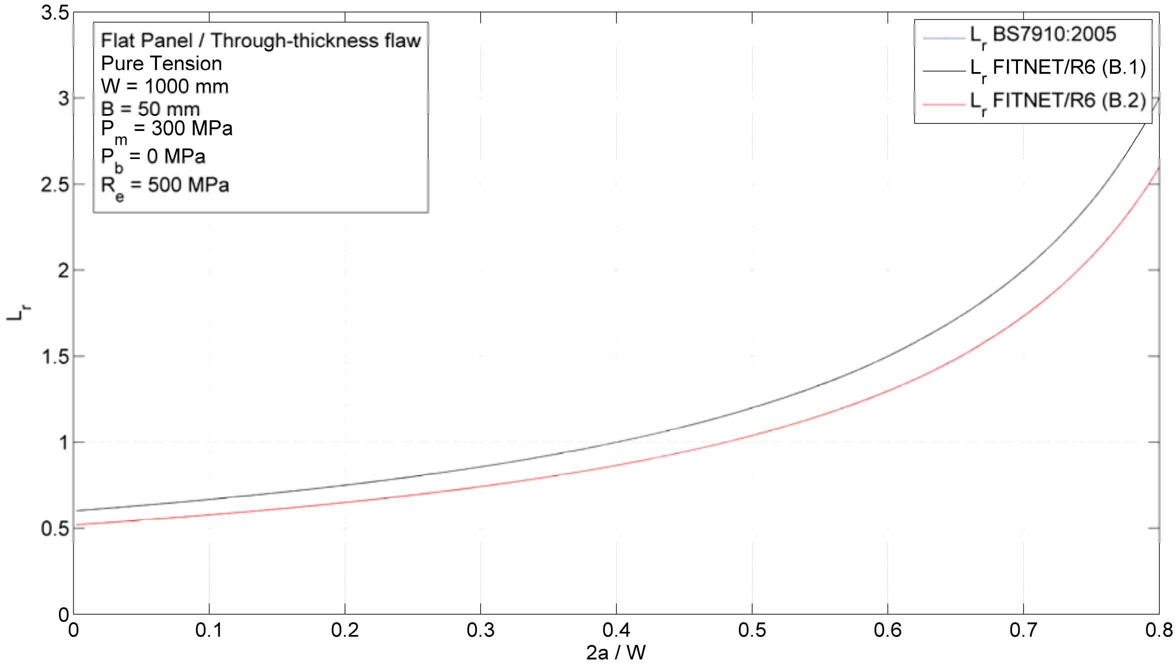

FIGURE 10: CENTRE CRACKED FLAT PLATE CONTAINING THROUGH-THICKNESS FLAW PURE TENSION – L_r

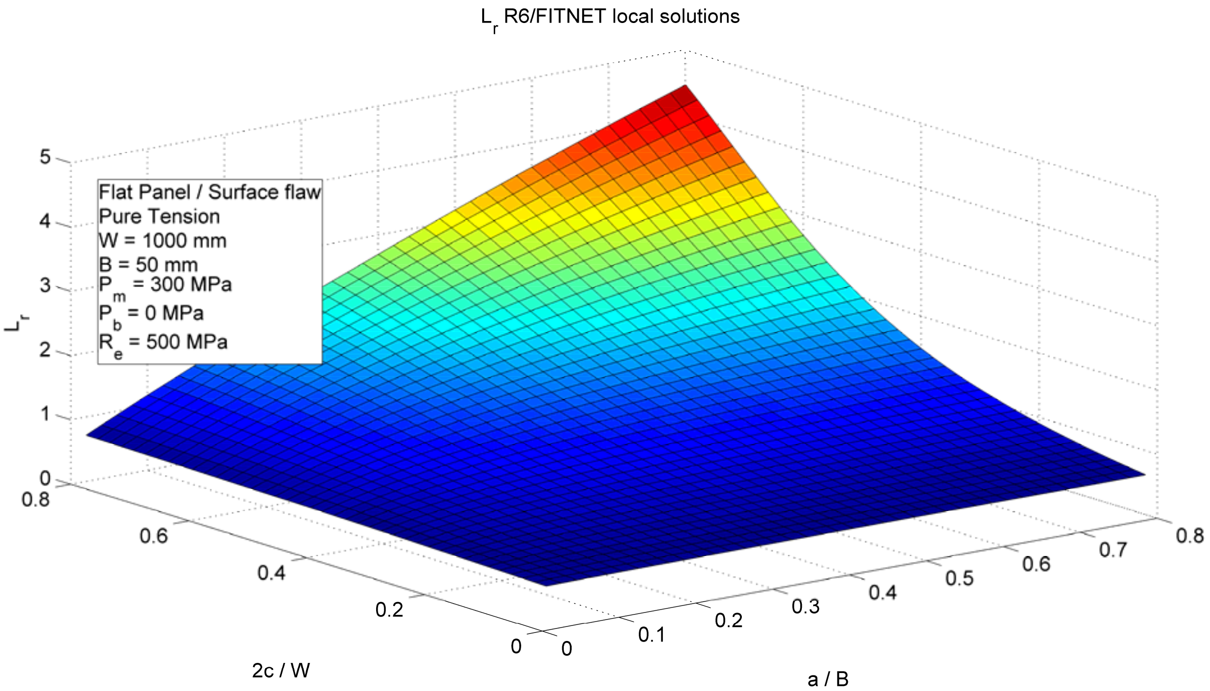

FIGURE 11: FLAT PLATE CONTAINING SURFACE FLAW – PURE TENSION LOCAL SOLUTION – R6/FITNET L_r SURFACE

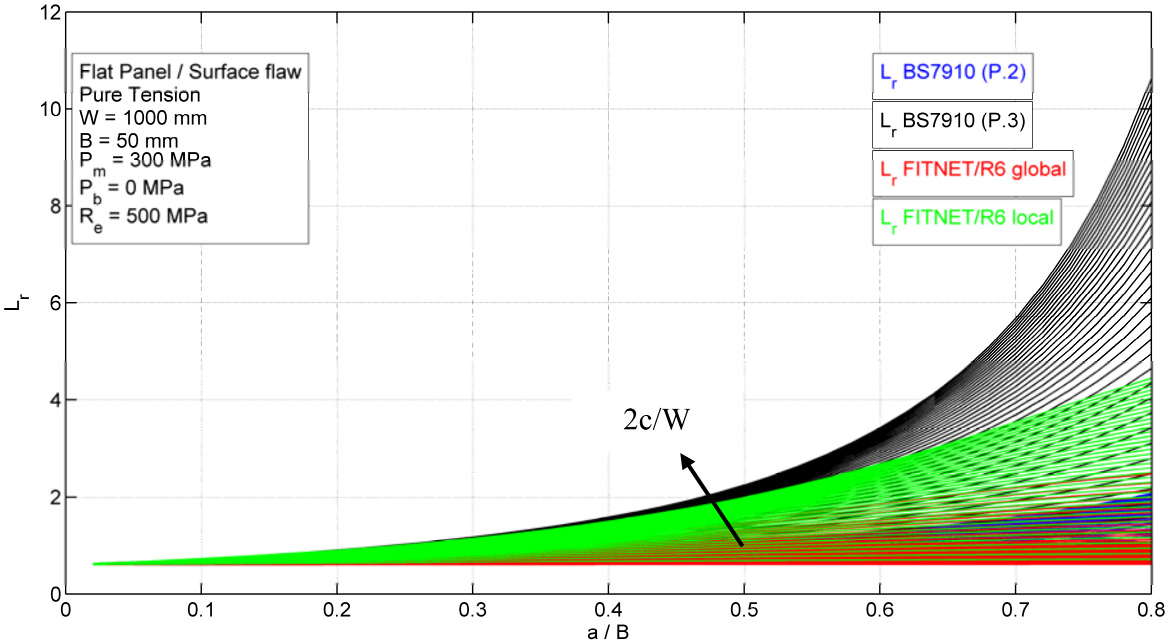

FIGURE 12: FLAT PLATE CONTAINING SURFACE FLAW PURE TENSION – L_r

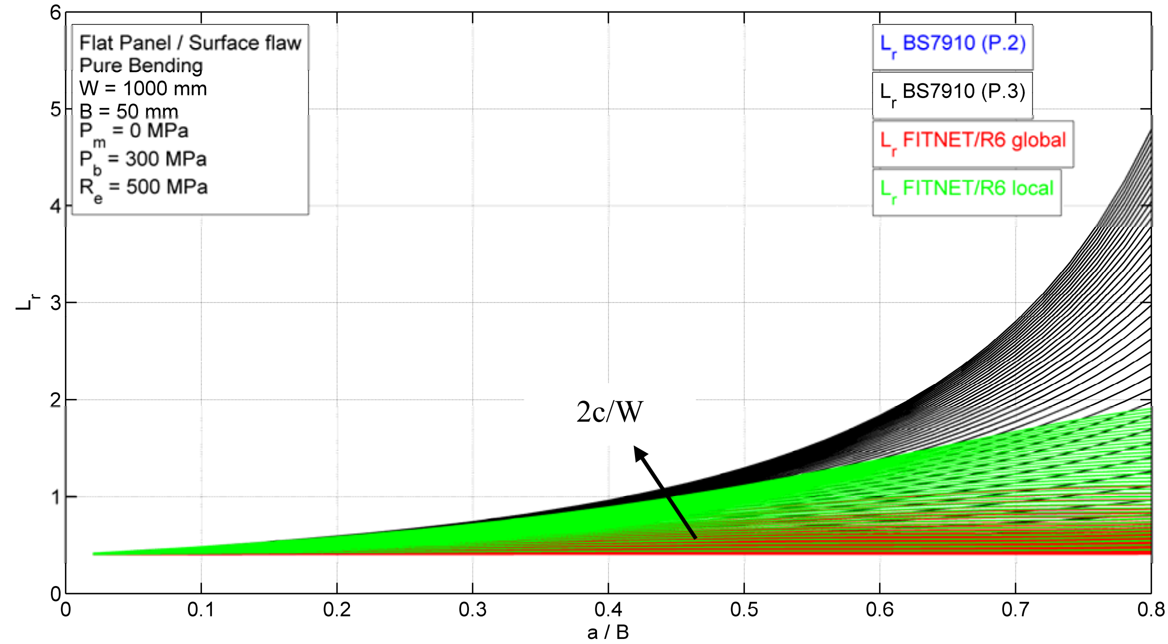

FIGURE 13: FLAT PLATE CONTAINING SURFACE FLAW PURE BENDING – L_r

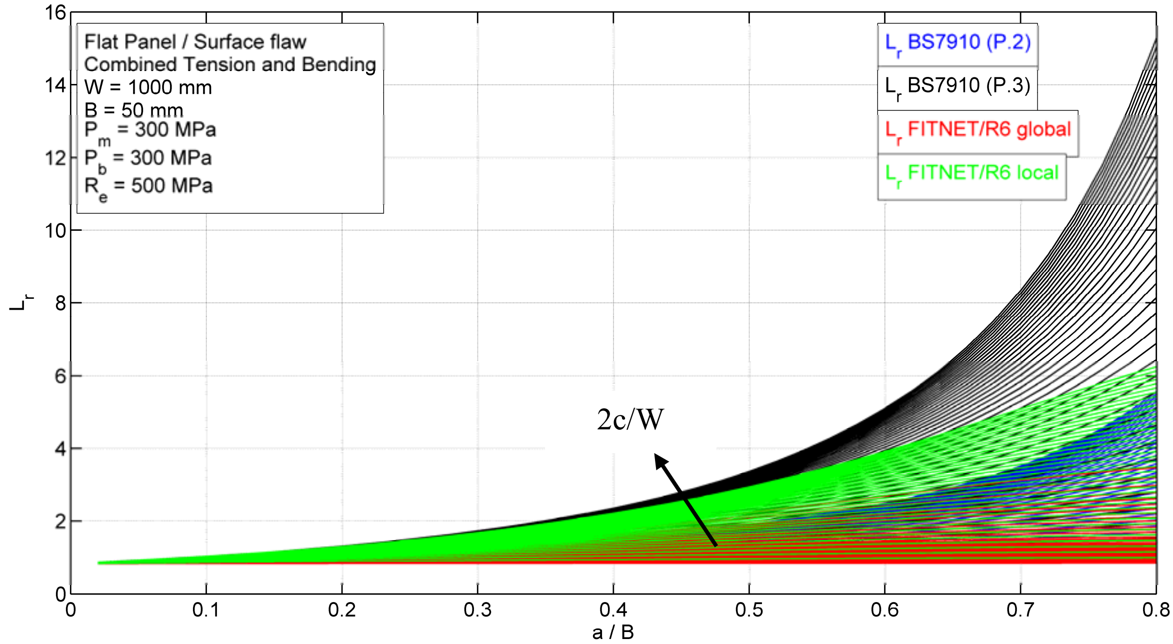

FIGURE 14: FLAT PLATE CONTAINING SURFACE FLAW COMBINED TENSION AND BENDING – L_r

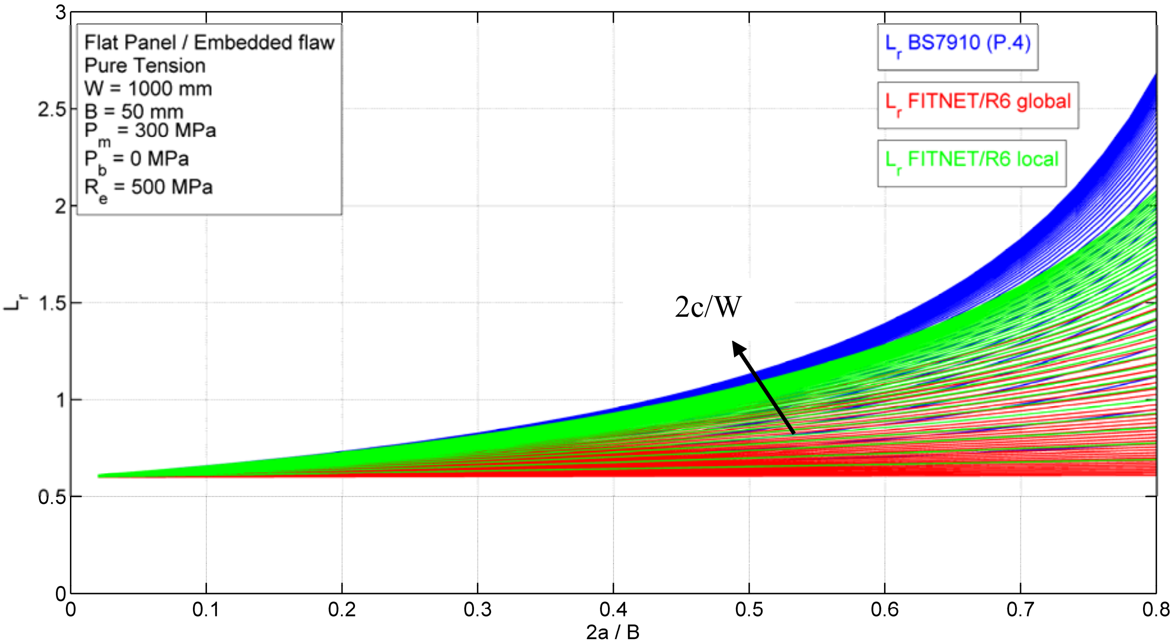

FIGURE 15: FLAT PLATE CONTAINING EMBEDDED FLAW PURE TENSION – L_r

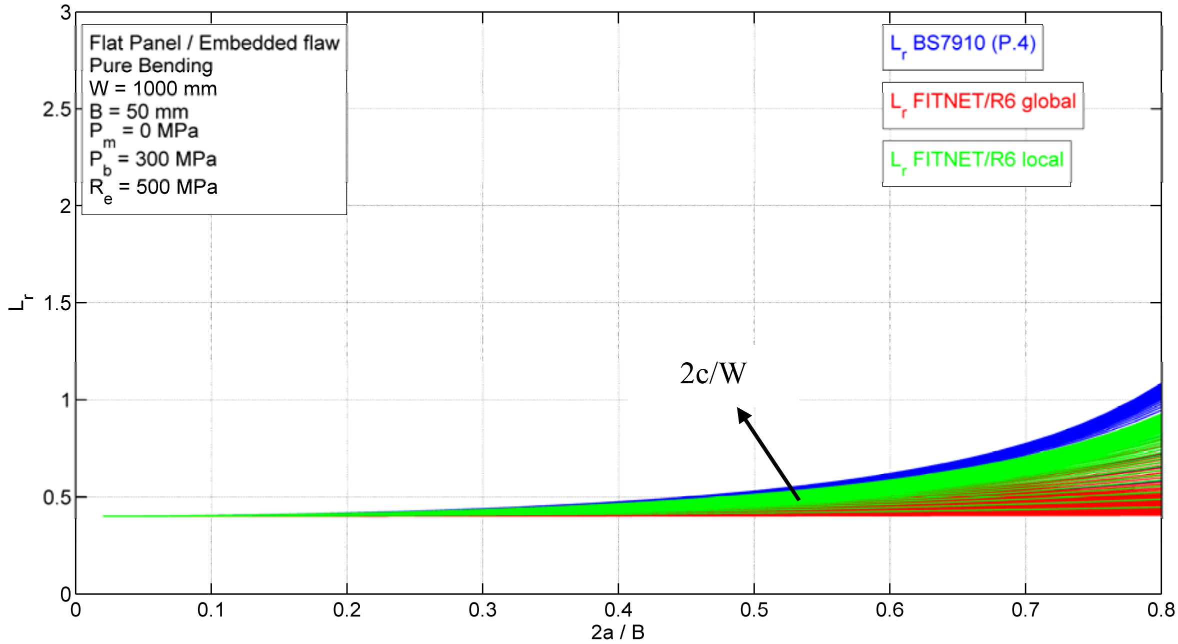

FIGURE 16: FLAT PLATE CONTAINING EMBEDDED FLAW PURE BENDING – L_r

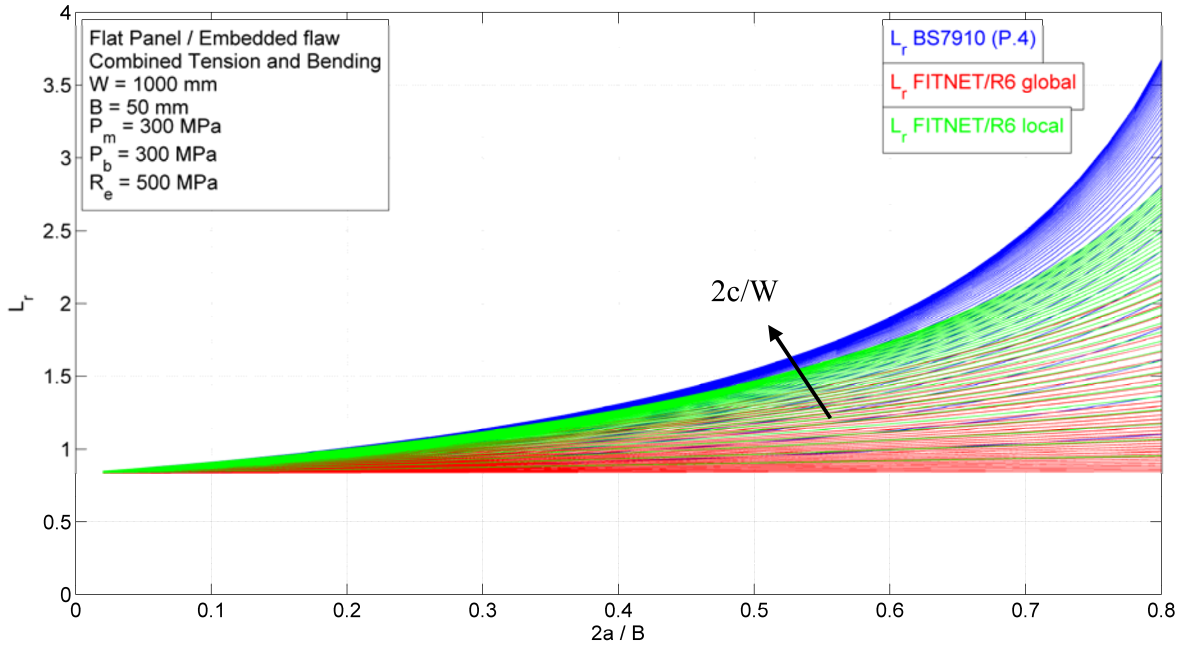

FIGURE 17: FLAT PLATE CONTAINING EMBEDDED FLAW COMBINED TENSION AND BENDING – L_r

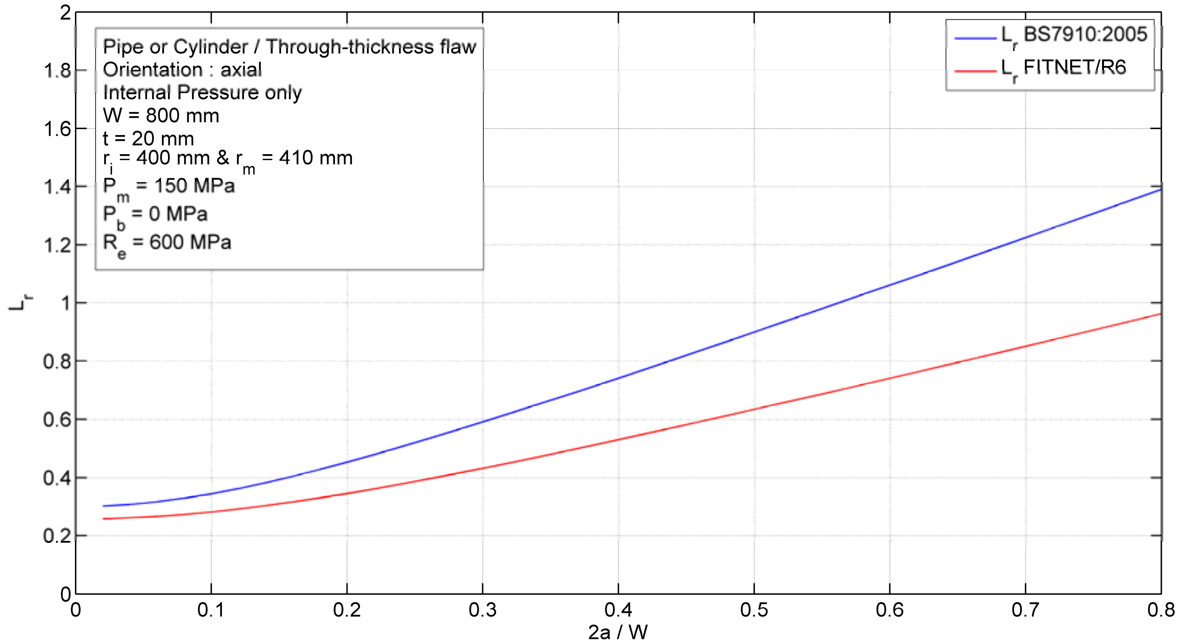

FIGURE 18: PIPE/CYLINDER CONTAINING AXIAL THROUGH-THICKNESS FLAW INTERNAL PRESSURE ONLY – L_r

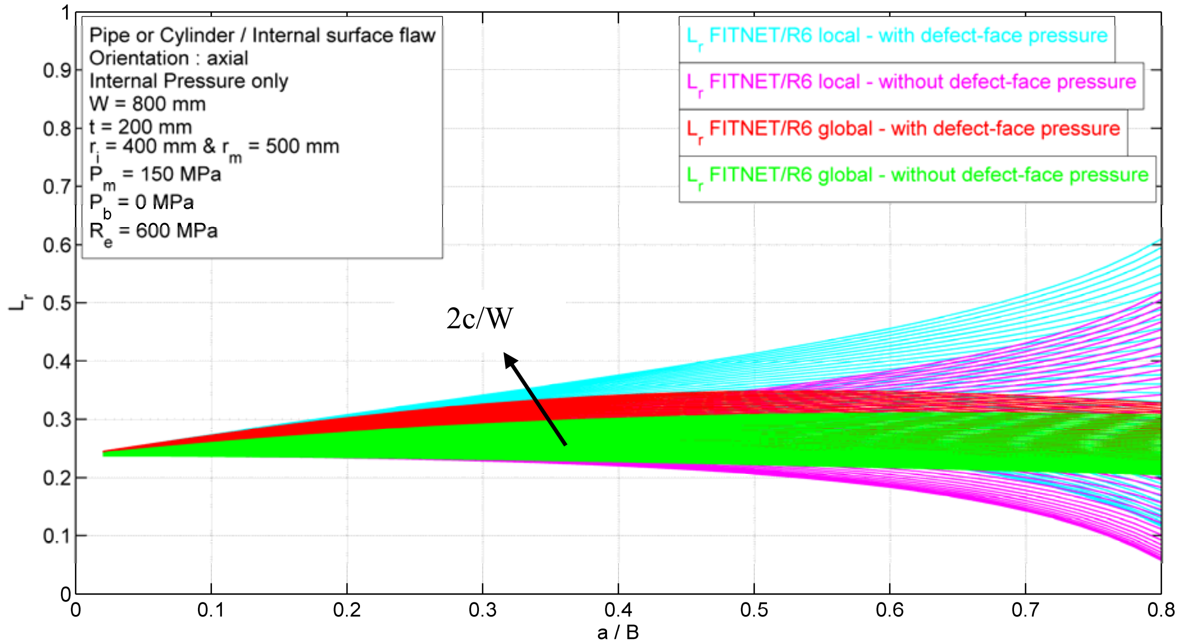

FIGURE 19: PIPE/CYLINDER CONTAINING AXIAL INTERNAL SURFACE FLAW INTERNAL PRESSURE ONLY – R6/FITNET L_r

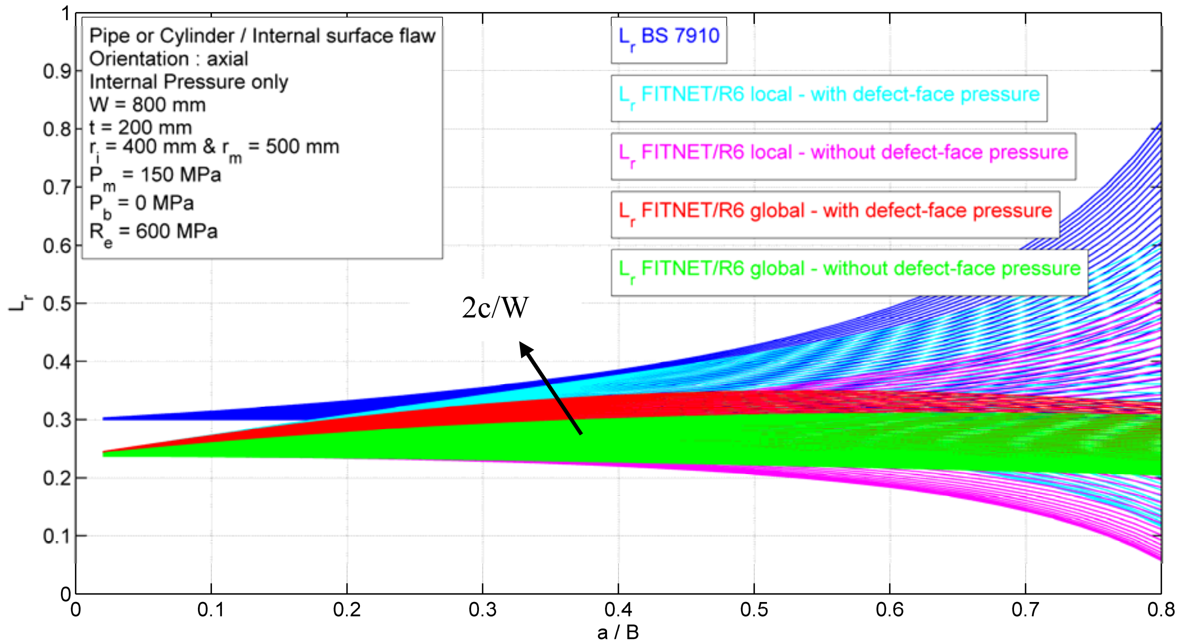

FIGURE 20: PIPE/CYLINDER CONTAINING AXIAL INTERNAL SURFACE FLAW INTERNAL PRESSURE ONLY – L_r

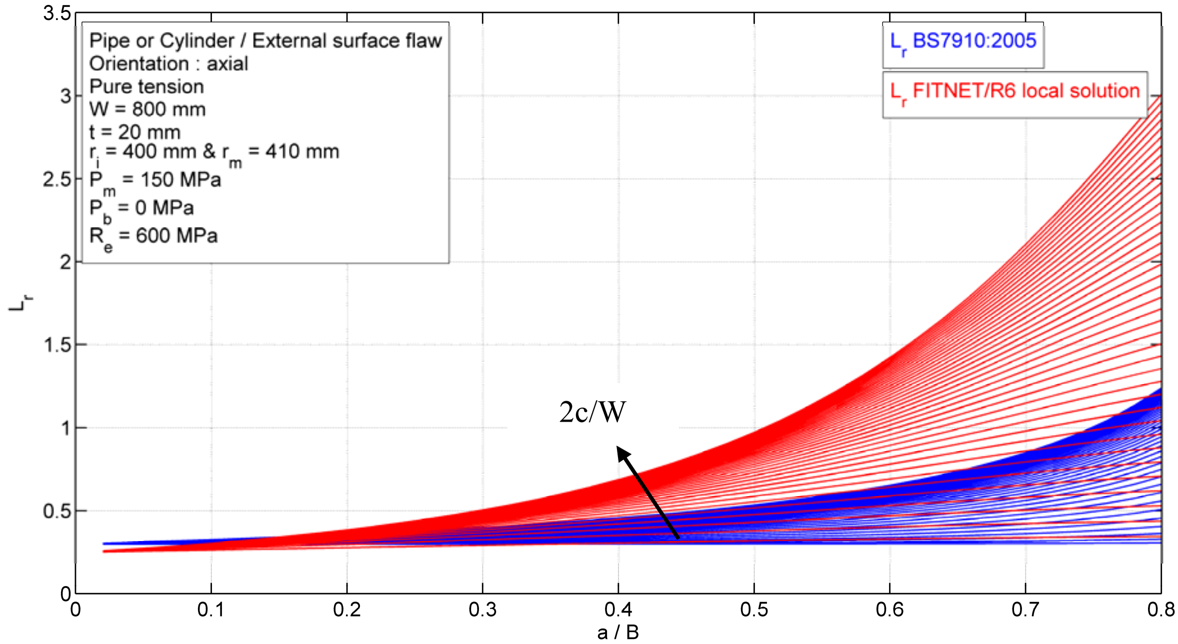

FIGURE 21: PIPE/CYLINDER CONTAINING AXIAL EXTERNAL SURFACE FLAW PURE TENSION – L_r (THIN-WALLED PIPE/CYLINDER)

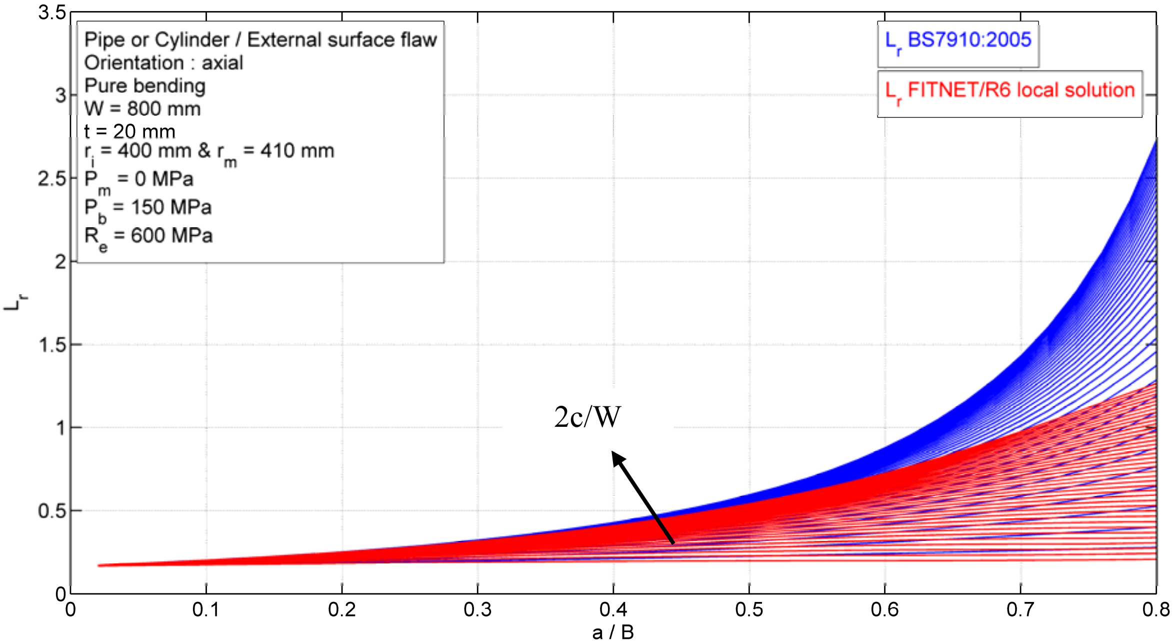

FIGURE 22: PIPE/CYLINDER CONTAINING AXIAL EXTERNAL SURFACE FLAW PURE BENDING – L_r (THIN-WALLED PIPE/CYLINDER)

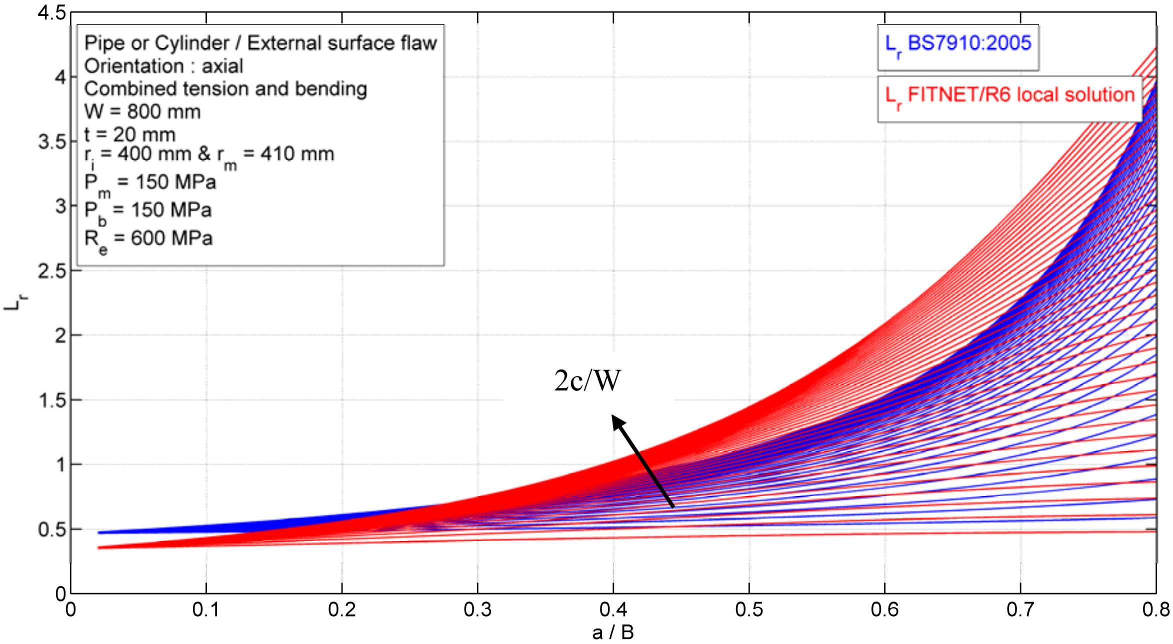

FIGURE 23: PIPE/CYLINDER CONTAINING AXIAL EXTERNAL SURFACE FLAW COMBINED TENSION AND BENDING – L_r (THIN-WALLED PIPE/CYLINDER)

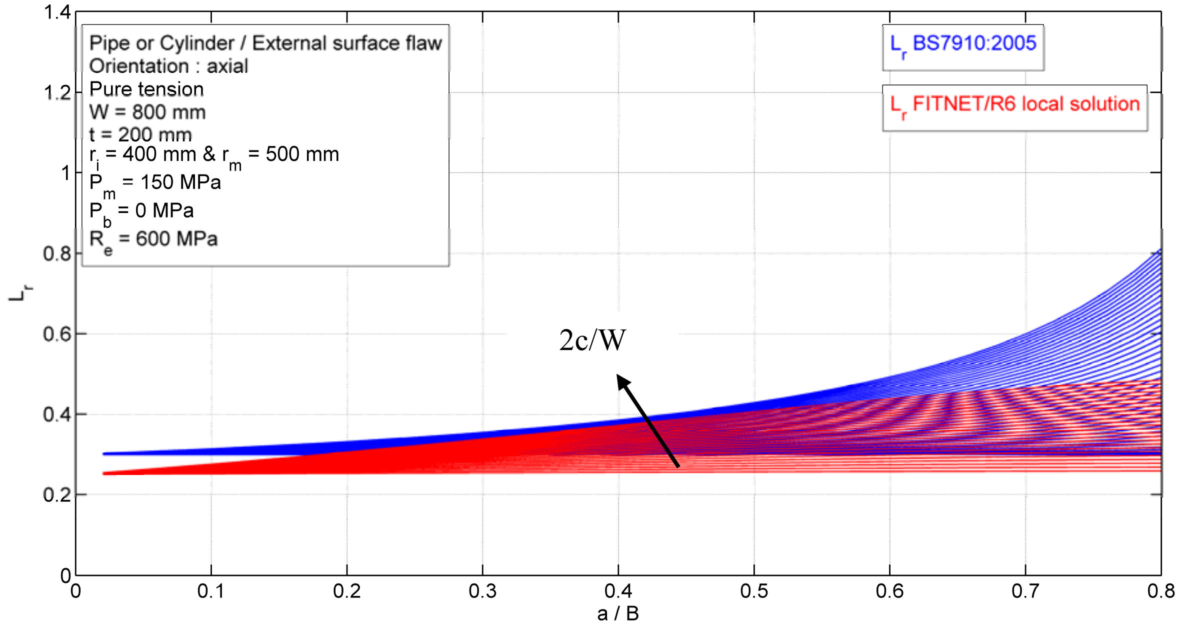

FIGURE 24: PIPE/CYLINDER CONTAINING AXIAL EXTERNAL SURFACE FLAW PURE TENSION – L_r (THICK-WALLED PIPE/CYLINDER)

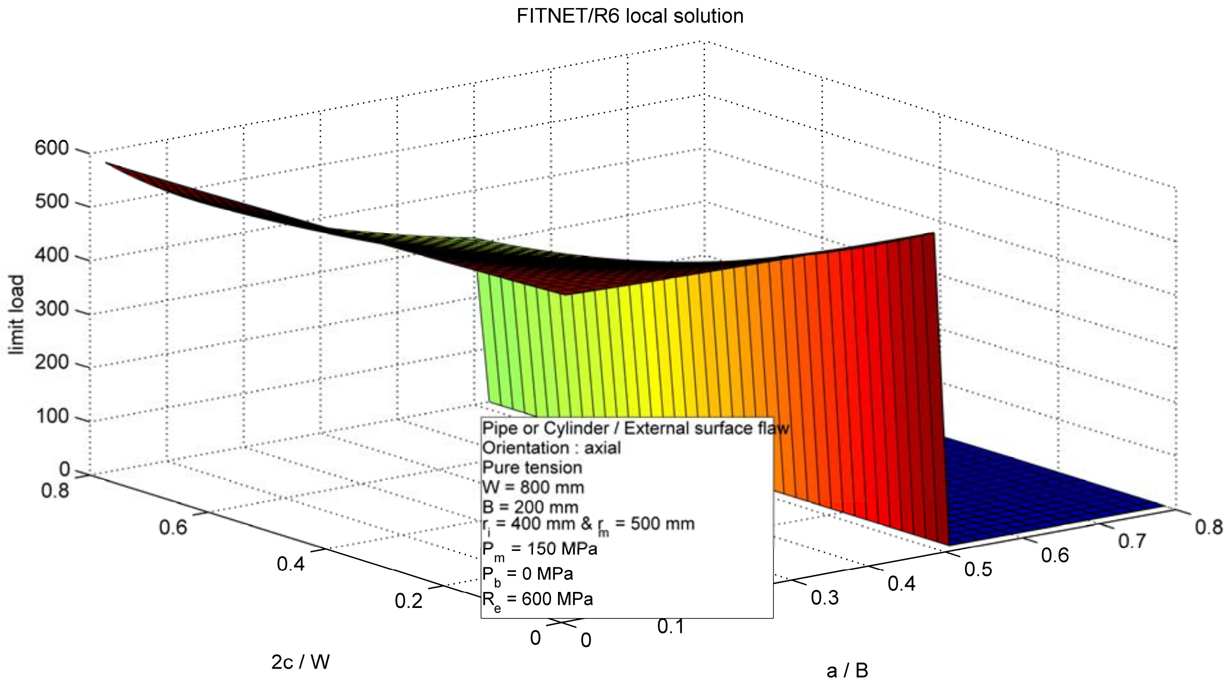

FIGURE 25: PIPE/CYLINDER CONTAINING AXIAL EXTERNAL SURFACE FLAW PURE TENSION – LIMIT LOAD SURFACE (THICK-WALLED PIPE/CYLINDER)

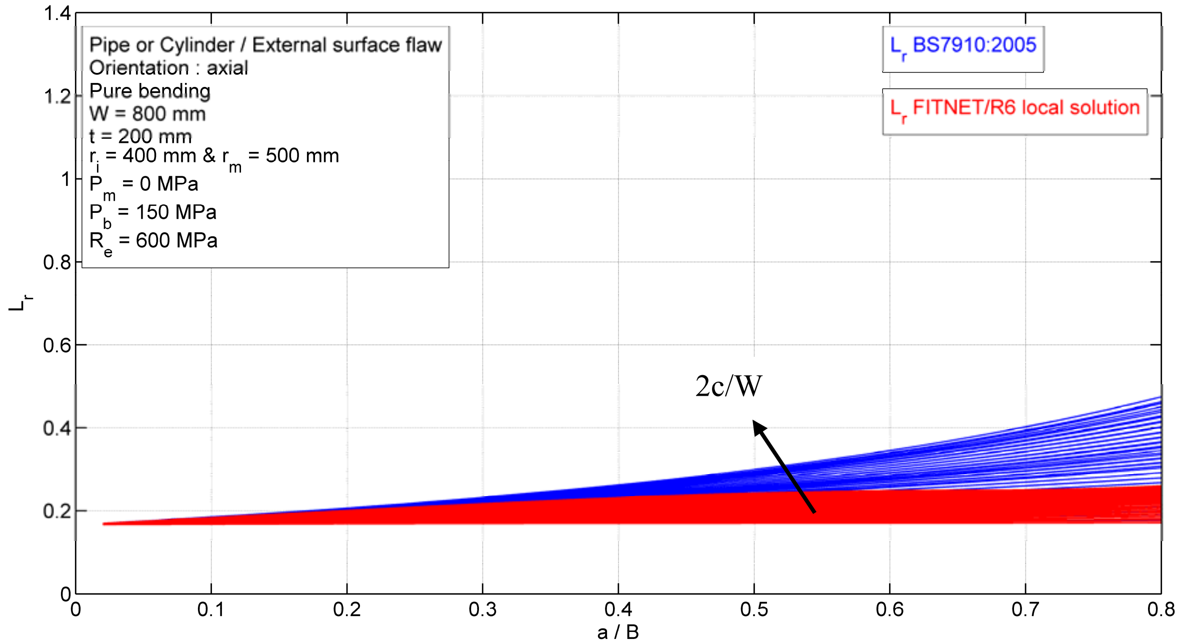

FIGURE 26: PIPE/CYLINDER CONTAINING AXIAL EXTERNAL SURFACE FLAW PURE BENDING – L_r (THICK-WALLED PIPE/CYLINDER)

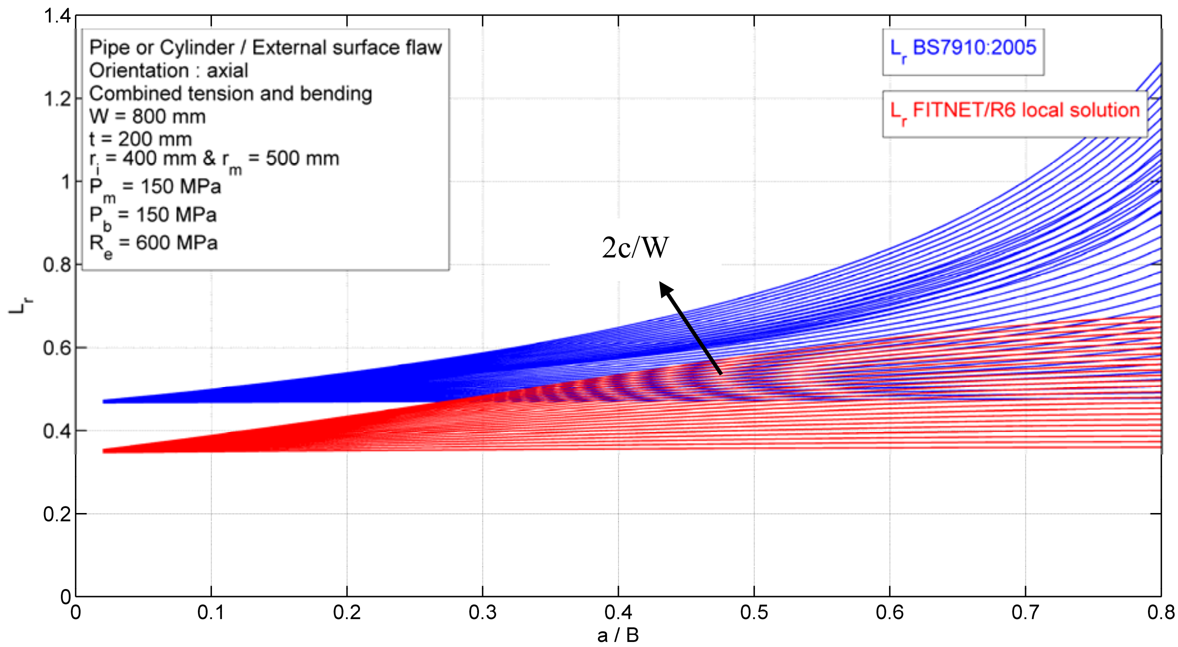

FIGURE 27: PIPE/CYLINDER CONTAINING AXIAL EXTERNAL SURFACE FLAW COMBINED TENSION AND BENDING – L_r (THICK-WALLED PIPE/CYLINDER)

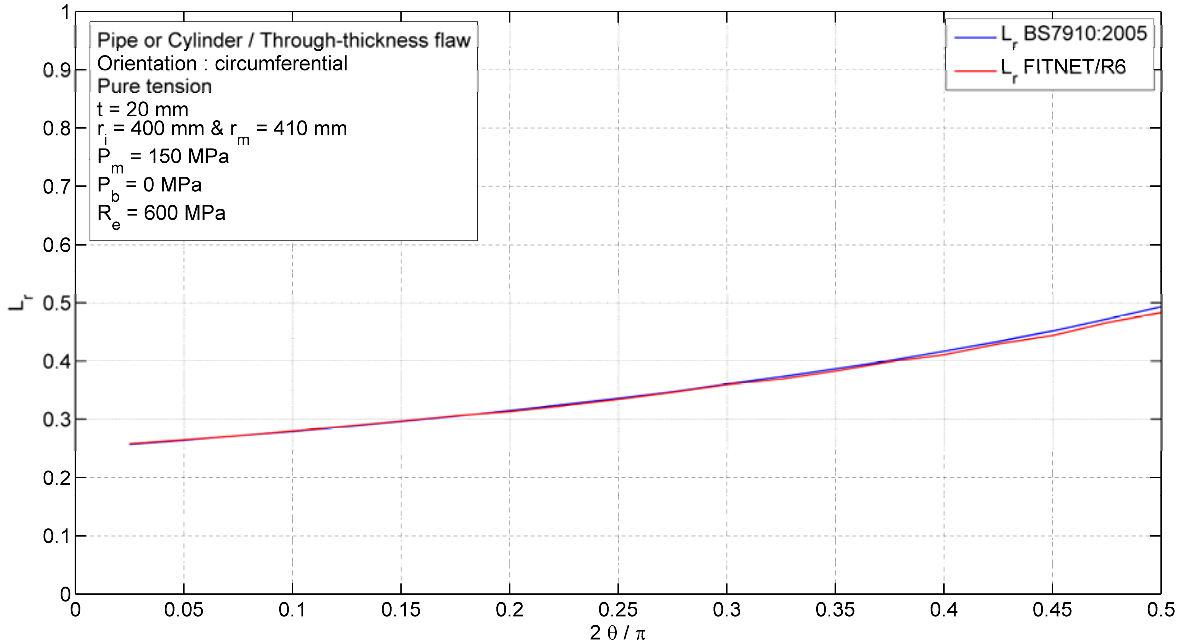

FIGURE 28: PIPE/CYLINDER CONTAINING CIRCUMFERENTIAL THROUGH-THICKNESS FLAW PURE TENSION – L_r (THIN-WALLED PIPE/CYLINDER)

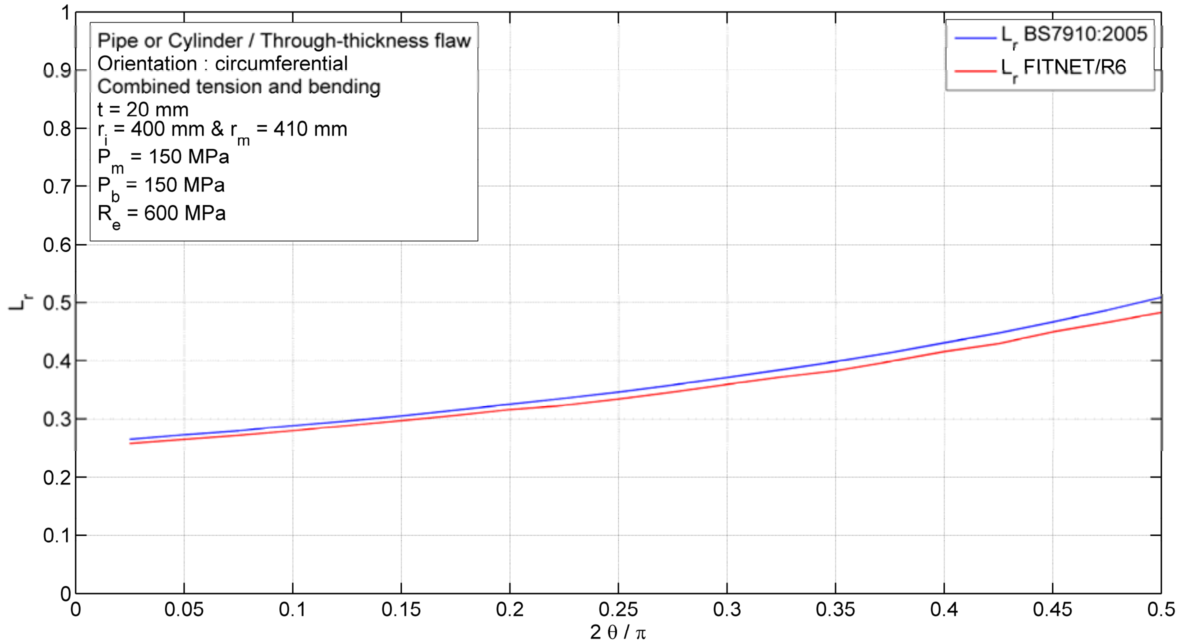

FIGURE 29: PIPE/CYLINDER CONTAINING CIRCUMFERENTIAL THROUGH-THICKNESS FLAW COMBINED TENSION AND BENDING – L_r (THIN-WALLED PIPE/CYLINDER)

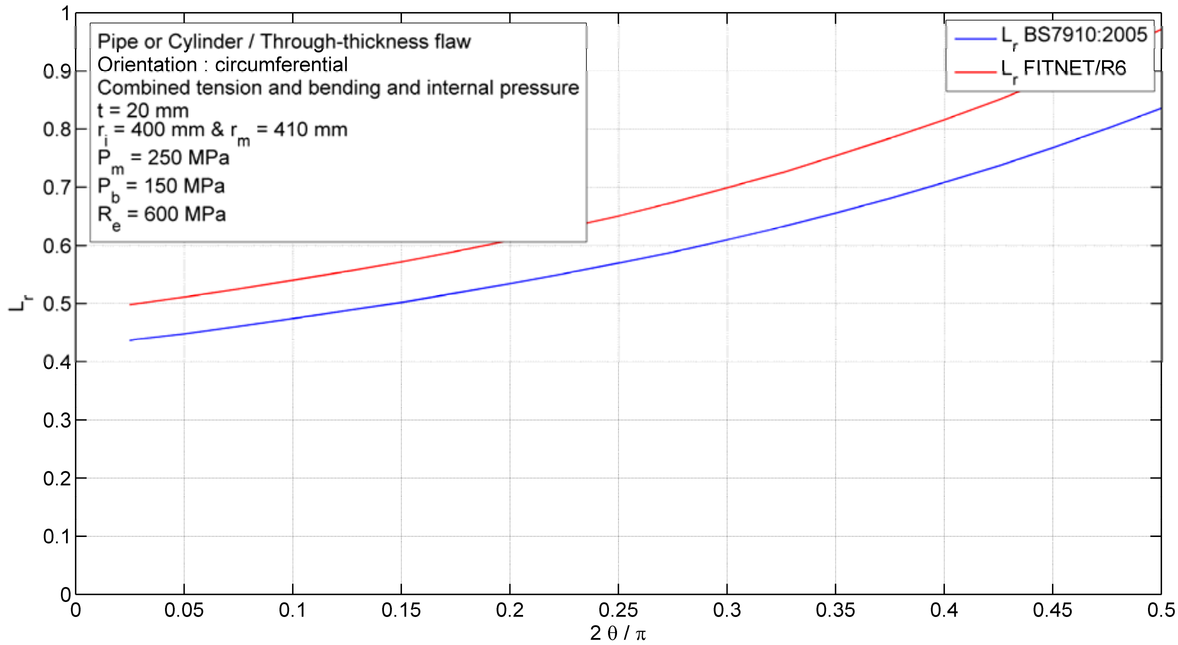

FIGURE 30: PIPE/CYLINDER CONTAINING CIRCUMFERENTIAL THROUGH-THICKNESS FLAW COMBINED TENSION AND BENDING AND INTERNAL PRESSURE – L_r (THIN-WALLED PIPE/CYLINDER)

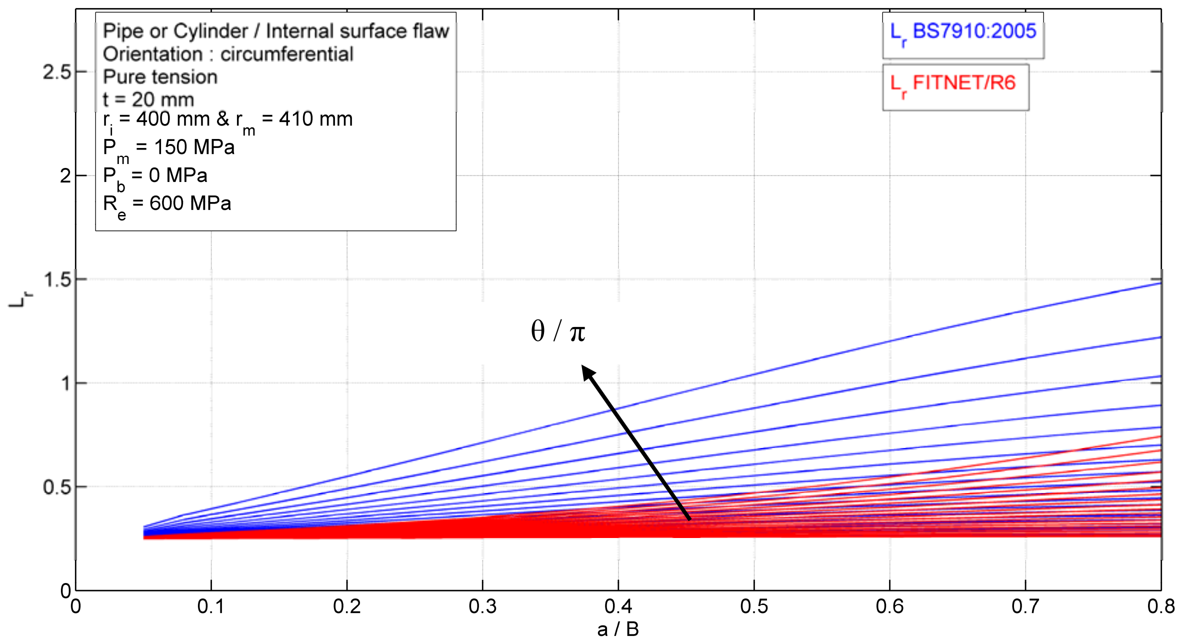

FIGURE 31: PIPE/CYLINDER CONTAINING CIRCUMFERENTIAL INTERNAL SURFACE FLAW PURE TENSION – L_r (THIN-WALLED PIPE/CYLINDER)

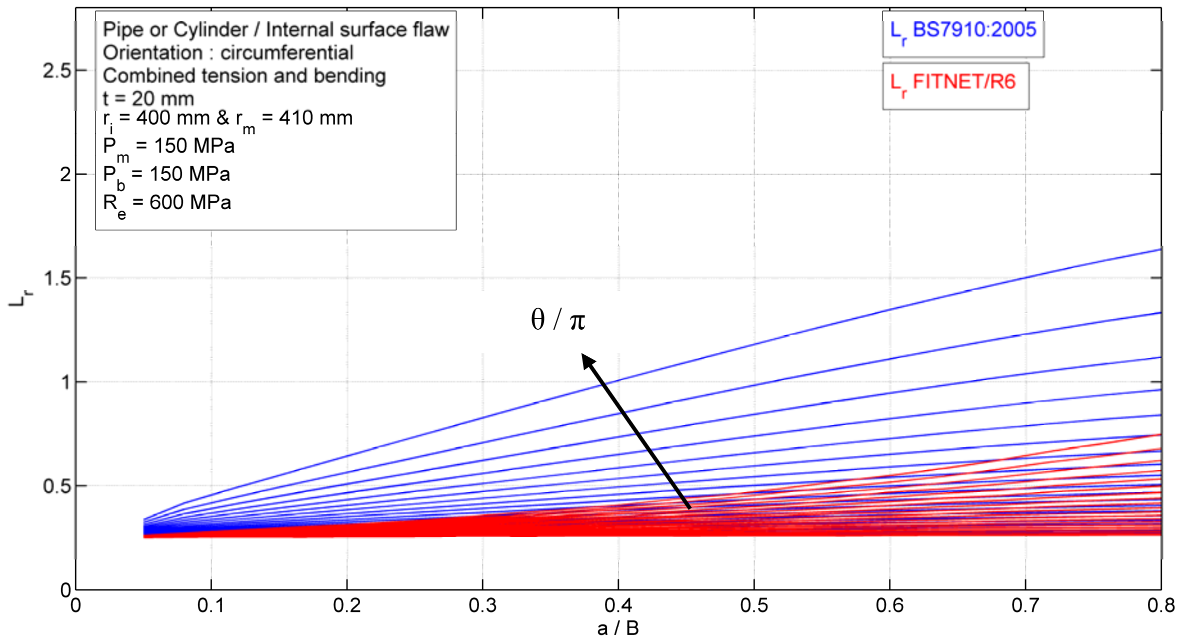

FIGURE 32: PIPE/CYLINDER CONTAINING CIRCUMFERENTIAL INTERNAL SURFACE FLAW COMBINED TENSION AND BENDING – L_r (THIN-WALLED PIPE/CYLINDER)

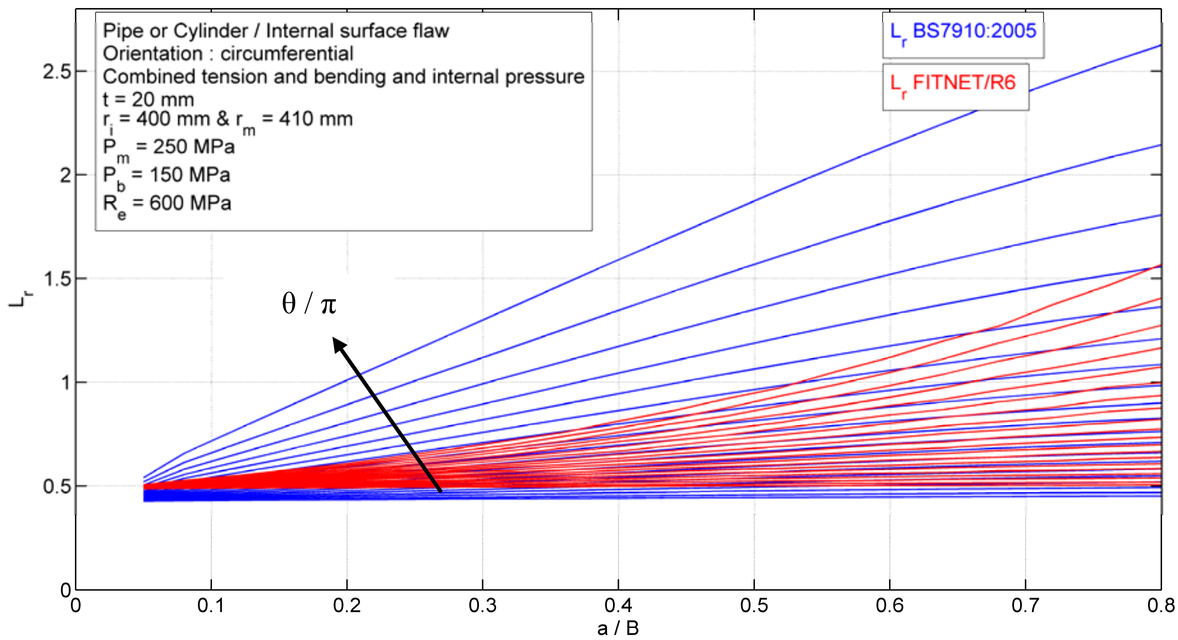

FIGURE 33: PIPE/CYLINDER CONTAINING CIRCUMFERENTIAL INTERNAL SURFACE FLAW COMBINED TENSION AND BENDING AND INTERNAL PRESSURE – L_r (THIN-WALLED PIPE/CYLINDER)